



Testing Classical Interactions between Finite Particles as a Model of Nuclear Structure

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Abstract

A finite dimensional model for the electron and proton has been used to compute nuclear properties such as: structure, binding energies, energies and rates of decay of radioactive isotopes.

Computations were conducted within the frame of classical electromagnetic interactions between *toroidal* electrons and protons of finite, fixed dimensions. Positions and orientations of each particle were allowed to vary using the *variational method*, until the minimum energy configuration was attained.

Nucleon shell structures were found to build from outer levels toward inner ones, with occupancies following the *magic numbers* so well known in nuclear physics. Neutrons were found to be formed via toroidal protons binding electronically and magnetically within toroidal electrons, which are significantly larger than the former.

Details are presented for ^{40}K as a model test case. Additional results are provided for several select radio nuclides having a diversity of nuclear structures. These calculations, although admittedly of questionable accuracy, do none the less appear to yield results which are in some 90% agreement with the experimental values, over the very limited number of examples tested.

Keywords

Nuclear protons and electrons, Nuclear particle binding, Radioactive decay energies and rates

Introduction

In this work nuclear particles are modeled according to the proposals of Bergman (1991) and Lucas (1996), in which all classical electromagnetic interactions are between electrons and protons producing neutrons and between protons and protons per se. Each particle is described having a *toroidal* geometry of electrostatic charge, negative and positive for electrons and protons respectively.

The accepted rest radii of the electron, $3.87 \times 10^{-13}\text{m}$, and the proton, $2.11 \times 10^{-15}\text{m}$, (Bergman, 1991) are for free, unbound particles. But in this study it was found that binding reduces the electron radius by two orders of magnitude and expands the proton radius by 183% (3.86×10^{-14}). If these optimized adjustments are not made, the computed nuclear binding energies are found to be about one hundred times smaller than required for acceptable values. In this paper, we will not address the question of whether these sizes agree with scattering experiment results.

These toroidal particles are so infinitesimally thin ($\approx 10^{-200}\text{m}$), that they may be regarded as electromagnetic static *current loops* of fixed dimensions, charges and magnetic moments. These current loops appear to be appropriate particle descriptions for the calculations presented in this paper.

Derivation of the model

An exact expression for all electro/magneto static interactions between toroidal electrons and protons (except for the self energies), which constitute the nucleon components of an atomic nucleus, has been derived by Eric Baxter for this study. The basic equation is the following

$$E = 1/2 \sum_{i,j} \iint \frac{\partial}{\partial \varepsilon_i} \frac{\partial}{\partial \varepsilon_j} \left\{ \frac{q_i q_j}{R_{ij}} - \frac{\vec{m}_i \cdot \vec{m}_j - 3(m_i \cdot \hat{r}_{ij})(m_j \cdot \hat{r}_{ij})}{R_{ij}^3} \right\} d\varepsilon_i d\varepsilon_j \quad (1)$$

Where E is the total nuclear binding energy less the self energy, i and j label the specific particles, ε_i and ε_j are the internal angular coordinates of each *loop*, q_i and q_j are electrostatic charges, R_{ij} the inter particle separations and m_i , r the particle magnetic moment and electric vectors respectively.

Integrations are carried out over all angular orientations within the boundaries of each *current loop*. The required numbers of nucleons are initially distributed within shells, using either the conventional *magic numbers* or the ring *model scheme* (Lucas, 1996). In the case of ^{40}K , for example, there are 19 protons for the atomic number plus 21 protons and

21 electrons forming neutrons. If *magic numbers* are employed for initial shell assignments, the occupancies are (from inner to outer levels) 2,8,10,20, protons and 1,5,10,5, electrons. But with the *ring model shell scheme*, the assignments are 8,32 protons and 5,16 electrons. In either case the electrons are paired off with protons to form neutrons.

While the methodology of the calculations is independent of initial particle assignments (since energy minimization via the variational approach will attain the correct final shell assignments), if the initial assignments are reasonably close to the final, there is a great saving on time constraints for minimization. It has been our experience that the *ring shell model scheme* is preferred for it always comes closest to the final shell occupancies. Perhaps this is owing to the fact that this scheme accurately reproduces reported nuclear spins of *all* isotopes for which values are listed; whereas, the *magic numbers scheme* which are correlated with the quantum mechanical model derived from nuclear wave functions, appear to have some 65-80% over all reliability in reproducing nuclear spins of *all* isotopes (Linde, 1990–1991; Lucas, personal communication, 2000).

All computations based on equation (1) were conducted with a computer program (PASCAL) written by Eric Baxter. The nuclear radii are given by $r=(1.2-1.3) A^{1/3}$, where 1.2 femtometers is preferred for mass numbers, A , $<\sim 200$ and 1.3 for $A>200$. This is used to locate the maximum shell position from the nuclear center, which is divided proportionally into segmented regions for accommodating the total numbers of nucleons within each designated shell. Of course the entire arrangement is minimized variationally to a final, minimum energy configuration. The order of filling is actually from the outer-most levels inward (contrary to extra-nuclear electrons which are filled from the inner-most shell outward). Hence, the outer-most nuclear shell contains the most energetically stable nucleons, while the least stable nucleons are contained within the last, inner-most shell.

Each nuclear particle (proton and electron) is specified by five coordinates; three positional and two angular. The three positional x, y, z spatial coordinate identify the location of each toroidal particle, while the angular coordinates ε_{ij} , in equation (1) are composed of two angular θ and ϕ coordinates specifying all tilt orientations of each *current loop*. All coordinates are allowed to fluctuate, while the total energy is minimized according to the variational principle, for which $\delta=0$ when the following conditions are satisfied

$$\frac{\partial E}{\partial x_{ij}} = \frac{\partial E}{\partial y_{ij}} = \frac{\partial E}{\partial z_{ij}} = 0 \text{ and } \frac{\partial E}{\partial \theta_{ij}} = \frac{\partial E}{\partial \phi_{ij}} = 0 \quad (2)$$

E is of course the minimum nuclear binding energy, as previously stated.

To obtain decay energies, say for β^- emission for example, an electron is removed from the least stable neutron and the total energy re-minimized as previously. For β^+ emission an electron is added to the least stable proton and the minimum energy recalculated. In the case of α^{2+} emission, 2 electrons and 2 protons are removed from their respective, least stable, shells and the total energy again re-minimized.

All calculated NBE (nuclear binding energies) for beta decay processes are reported in Table 1 together with experimental values. Although equation (1) is exact to the extent of what it entails, exact NBE values are not calculated for several reasons. In the first place, the self energies of the particles are not included in this model. Secondly, it is likely that the *rings* experience polarization effects, though relatively small, are not necessarily negligible. Thirdly, all computational routines require approximations of varying degree and are not truly “exact,” in the literal sense of the word. While the inclusion of self energies is feasible, accounting for polarization effects is not. Attempts to account for all contributing factors would enormously complicate the integrations and convergence criteria. Of course a MonteCarlo routine could be invoked, which would improve the likelihood of success with more refined calculations, but this requires more computing capacity than what is available to us. Certainly anyone who is interested in pursuing this approach is indeed encouraged to do so.

Fortunately, accurate NBE are not essential for obtaining reasonably accurate decay energies (as shown in Table 1), which depend on differences in NBE and not their accurate values. Whatever discrepancies in NBE are present in one parent isotope are also present to the same extent in its daughter isotope, for which the differences cancel upon evaluating the decay energy. Note that the decay energies presented in Table 2 are accurate within 90–99% of the accepted values.

Results and Discussion

Initial test calculations of NBE and decay energies were made on, ^8Be , ^{24}Na , ^{24}Mg (stable) and ^{40}K as listed in Table 2. We will use ^{40}K to demonstrate details of these calculations. It was found that ^{40}K proved to be a most interesting case, calculated to have (surprisingly) *two* potential minima of different energies.

The ^8Be isotope decays nearly spontaneously into two α -particles, while ^{24}Na is a β -emitter producing one stable product, ^{24}Mg . However, ^{40}K decays by β^- , electron capture and by positron (β^+) emission. The

Table 1. Ring model nucleon shell structure for ^{40}K .

Shell Number	1		2		3		4			
Nucleons:	p ⁺	n ⁰	p ⁺	n ⁰	p ⁺	n ⁰	p ⁺	n ⁰	Spin	E (β ⁻) Cal. ^a
Number of Nucleons	-	1	3	4	-	-	16	16	2	3.180
	-	-	3	5	-	-	16	16	4	1.312

a. Calculated β-decay energies in MeV.

β-minus decay process produces ^{40}Ca as the stable daughter product and accounts for the major portion of the decay mechanism. The electron capture decay process produces the noble gas ^{40}Ar as its daughter and is the one employed in the radiometric dating of rocks. Of course argon is a volatile gas and in our opinion cannot possibly be a reliable chronometer for accurate radiometric dating.

These calculations have provided two spin states for ^{40}K which are not at the same energy. These appear as two minima in the binding energy profile for ^{40}K . The spin 2 state is 1.86MeV greater than the spin 4 minimum based on: Calculated BE (spin4)=297.40 MeV; BE (spin2)=299, vs. ^{40}Ca BE=296.08MeV. The decay from spin 4 is 1.32MeV while that for spin 2 is 3.18MeV. Experimental data (Lederer & Shirley, 1978) show a spin 3 excited states at 0.0296MeV and a spin 2 excited states at 0.8001MeV above the ^{40}K ground state. However, these calculations could not detect this spin 3 excited state, since a binding energy difference energy of 0.03 is well within the error limits (about 90 % of a total BE) of reliability

in these calculations. Similarly, an energy of 0.8MeV above the ground state is still not likely to be clearly represented as a minimum in the binding energy profile of ^{40}K . These are clearly limitations within the current capability of this model.

It should also be noted that a gamma decay (with gamma rays of 1.86MeV) from the calculated higher energy spin 2 state (to the observed higher energy spin 4 state) should compete with the β-minus decay to ^{40}Ca . A γ-decay with ΔI=2 (no parity change) would probably dominate with an estimated half life of the order of picoseconds.

We have found (unpublished results) that the decay energies of radioactive potassium isotopes exhibit a linear relation to the log of the decay constants, log λ, according to Sargent's Rule established in the early 1900s (Rutherford, 1930). This linear relation has been reanalyzed in terms of nuclear spins and the slopes of the curves found to be dependent upon even or odd spins. The spin states of ^AK , A=42, 44, 46, 48, all have spin 2 states (computed by us and reported from experiment), but ^{40}K was found to have two different spin states. The distributions of nucleons in ^{40}K according to the *ring shell model scheme* are presented in Table 1. One state of spin=4 has a decay energy of 1.312MeV, which is the only one reported experimentally. But the other state of spin=2 is clearly evident from the computed energy minima (which has been checked and reproduced three times), and has a decay energy of 3.180MeV.

Table 2. Finite toroidal particle calculations for select radioisotopes.

Isotope	Z ^a	A ^b	NBE ^c		Decay Mode	Decay calc.	Energy exp ^d .	Half Life	
			calc.	exp.				cal.	exp ^d
Be	4	8	53.7	56.5	2α	0.051	0.04	~10 ⁻¹⁶ -10 ⁻¹⁷ s	1.7 × 10 ⁻¹⁶ s
Na	11	24	174.2	193.5	β ⁻	5.67	5.51 (4.91) ^e	1 da	0.63 da
Mg	12	24	180.5	198.3	S ⁻	-	-	-	-
K	19	40	297.4	341.5	β ⁻	1.3	1.32	1.3 × 10 ⁷ yr	1.3 × 10 ⁹ yr
					(β ⁺)	1.5	1.50)		
			299.4	-	β ⁻	3.18 ^g	-	21hr	-
K	19	42	316.3	359.2	β ⁻	3.50	3.52	12hr	12.2hr
K	19	44	333.6	376.1	β ⁻	5.9	5.66	23min	22.1min
K	19	46	344.1	391.9	β ⁻	7.11	7.72	120s	107s
K	19	48	376.1	416.0	β ⁻	11.4	(12?) ^h	7.2s	6.8s
Ca	20	40	296.1	42.1	S ^f	-	-	-	-
Th	90	231	1559.6	1760.3	β ⁻	0.4	0.389	24hr	25.2hr
Th	90	234	1589.3	1777.7	β ⁻	0.3	0.270	7.2s	6.8s

a. Atomic number

b. Mass number

c. MeV units; data from Wapstra & Grove (1971).

d. MeV units; data from: Linde, D. R. (1990–1991).

e. Another value in the literature (ref. in d. above)

f. Stable

g. Calculations yield two energy minima not reported in the literature

h. Reported value uncertain

It was found that the most accurate relationship between $\ln \lambda$ and E (decay energy) for the even mass numbers of K radioactive isotopes, is the following quadratic equation

$$\ln \lambda = \sum_{i=0}^2 A_i E^i \quad (3)$$

For which: $A_0 = -18.171$; $A_1 = 2.463$; $A_2 = -0.0931$, with a least squares accuracy of $99.12 \pm 0.27\%$.

For ^{40}K in spin state 2, $E = 3.180 \text{ MeV}$, $\lambda = 1.3 \times 10^{-5} \text{ s}^{-1}$, $t_{1/2} = 21 \text{ hrs}$; while for spin state 4, $E = 1.320 \text{ MeV}$, for which there should be no fit to equation (3) since it has been derived for K spin states of 2. There is a reasonable fit for ^{40}K spin 4 from the following expression: $\ln \lambda = 16.0 \ln E - 38.5$. This was derived from the very limited data available (three data points) for β decay from a spin 4 ground state. The average error is $12.5(+12/-6)\%$ in $\ln \lambda$. Hence, for ^{40}K spin 4, λ (calc) = $1.7 \times 10^{-15} \text{ s}^{-1}$ versus $1.7 \times 10^{-17} \text{ s}^{-1}$ reported. The three data points are the spin 4 ground state of ^{40}K , the isomeric level in Yttrium-98 (spin 4, half life 2.1 s, decay energy 9.8 MeV) and the ground state of Aluminum-34 (spin 4, half life 0.060 s, decay energy 17.1 MeV). The spin 4 values of the ^{98}Ym and the ^{34}Al are not in older compilations of data, but are the results of more recent measurements (*CRC Handbook of Physics and Chemistry*, 2005; Baumann et al.; 1989; Nummela, et al, 2001).

Equations like that of (3) have been derived for various β and α (not reported here) decay processes, for purposes of obtaining λ values from computed decay energies.

Applications of the *toroidal ring model*, as described in detail for ^{40}K , have also been made on the β decaying isotopes ^{231}Th and ^{234}Th . These results are contained in Table 2.

Conclusion

Although only a limited number of examples have been tested, it indeed appears from the consistency in results that the Bergman-Lucas *toroidal ring model* of the electron and proton is an adequate and reliable basis for calculating nuclear structure, including

binding and radioactive decay energies. However, at this point in time, accuracy in calculating nuclear binding energies is lacking, since self energies (plus some higher order refinements) have not been incorporated into the model. Hopefully these adjustments will be made in the near future, if not by us then by someone else with interest in this project.

It is our opinion that by testing a new approach toward modeling nuclear structure from a classical electrodynamics basis rather than quantum mechanical, that the causal nature of finite particle interactions at the nuclear level is sufficient without invoking the need of nuclear wave functions and the *strong force vs. weak force* concepts.

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