



Studies of the Dependence of Nuclear Half-Lives on Changes in the Strength of the Nuclear Force

Eugene F. Chaffin, Ph.D., Bob Jones University, 1700 Wade Hampton Blvd., Greenville, SC 29614

Abstract

Nuclei which are of importance in radioisotope dating have very long half-lives, and calculations show that they are vulnerable to changes in the strength of the nuclear force. Their half-lives can change drastically. Although the “weak force” is the one responsible for beta-decays, the decay process is nonetheless very sensitive to the strength of the strong force gluing the nucleus together. In this paper, various possible sensitivities of the half-life for nuclear decays are investigated. In recent years, nuclear phase changes, such as the onset or loss of pairing interactions, or the shape transitions such as triaxial to oblate spheroidal and prolate spheroidal, have been a topic of interest among physicists. The pairing interactions, protons to protons or neutrons to neutrons have been found to disappear at high spin or at nuclear “temperatures” of a few tenths of an MeV. We investigate whether the change in nuclear force strength could cause breaking of the pairing bonds, hence leading to the possible loss of superfluidity or to mixed-phase nuclei. Quantum mechanical calculations are presented linking changes in various factors in alpha- and beta-decays to variation of the half-life. Tunneling processes, including nonlinear tunneling mechanisms, are investigated. According to modern theory, the W-particle has a mass-energy of 80.4 GeV and briefly enters the beta-decay process as a virtual particle leading to the emission, say in the beta-minus case, to an electron and an antineutrino. Calculations are given showing the sensitivity of this process to masses of the particles and other quantities which would be influenced by strong force variation. We discuss the linkage between various quantities and mechanisms by which small changes could possibly lead to large changes in the half-life.

Keywords

Radioisotope dating, Nuclear force, Forbidden decays, Phase change, Pairing

Forbidden Decays and Radioisotope Dating

Beta-minus decay, according to modern ideas, proceeds when one of the down quarks which make up a neutron emits a W^- particle. The down quark is thereby changed into an up quark, which also changes the neutron into a proton. Since the rest energy of the W^- particle is 80 GeV, which is more than the available energy, the W^- is a virtual particle and cannot escape but decays into an electron and an antineutrino.

Fermi (1934) did the first significant work on the theory of β -decay. He derived an expression for the decay rate of a nucleus. One factor in Fermi’s equation depended on the square of the modulus of the “matrix element” for a transition between the quantum state of the parent nucleus and the quantum state of the daughter nucleus. If a capital letter I is used for the quantum number of the total spin of the nucleus, Fermi’s matrix element was zero unless there was no parity change and

$$\Delta I = 0 \quad (1)$$

This equation is an example of a “*selection rule*.” If the selection rule was not satisfied, it did not mean that the transition rate was zero, but only that the matrix element would be very small and the corresponding half-life would be very long. Later, other types of matrix elements were found to also contribute (Burcham, 1963, p.607), leading to the so-called Gamow-Teller selection rules:

$$\Delta I = \pm 1, 0, \quad (2)$$

and no parity change, and also the transition $I=0$ to $I=0$ is not allowed. For both the Fermi and the Gamow-Teller selection rules, the *parity* of the nuclear state must not change during the transition, where parity is either positive or negative and specifies how the state changes during an inversion $\vec{r} \rightarrow -\vec{r}$. Transitions which obey the selection rules are said to be *allowed*, those which do not are *forbidden*.

It seems that radioisotope dating of rocks using β -decay is always done with isotopes which decay

via “forbidden decays.” Forbidden decays are not impossible ones but they are of much lower probability than “allowed” or “superallowed” decays. The word “forbidden” is here borrowed from its usage in atomic spectroscopy, where a transition between two electronic states of an atom is “allowed” if certain “selection rules” for the changes in the quantum numbers n , ℓ , j , μ , m_ℓ , m_s are obeyed, corresponding to so-called electric dipole transitions. However, just because a transition is not allowed does not mean that it never occurs. Higher-order processes such as “magnetic dipole” and “electric quadrupole” transitions may be possible, although at a much reduced rate. In the case of nuclear energy levels, there are selection rules operable in the β -decay transitions which are of interest. For the mathematically inclined it means that the matrix elements involve a different operator, but these matrix elements are small and do not become important unless the normal matrix elements vanish. But in the case of radioisotope dating we are usually using decays of this type because otherwise the half-life would not be very long. Nuclei with “allowed” β -decays invariably have a relatively short half-life and hence are not often used in radioisotope dating. A second factor is that the decay energy is usually small for decays of this type.

In the 1940s, before the theory was very well developed, the classification of transitions as “2nd forbidden,” “allowed,” etc. was usually done on an empirical basis, by looking at various graphs involving so-called “ft values” (Alburger, 1950; Berenyi, 1968; Brodzinski, and Conway, 1965; Konopinski, 1943; Konopinski & Uhlenbeck, 1935, 1941; Sastry, 1969). Back then, ^{14}C was thought to be a forbidden transition, but now we know that it is an allowed transition with a nuclear spin change of +1 and no parity change. On the timescale of interest here, the half-life of ^{14}C is also relatively short, at 5,715 years (Parrington, Knox, Breneman, Baum, & Feiner, 1996).

The theory of forbidden β -decays is discussed in nuclear physics textbooks, or in sources such as Behrens and Bühring (1982) or Konopinski (1943). In the limit of small decay energy Δ , where “small” is relative to a unit such as the MeV for nuclei, the fraction of all the radioactive atoms decaying per unit time, called the decay constant, is given by (Dyson, 1972):

$$\lambda = \frac{0.693}{T_{1/2}} = \begin{cases} G_F^2 K \Delta^{L+3} & Z \text{ small} \\ G_F^2 \Delta^{2+(1-\alpha^2 Z^2)^{1/2}} & Z \text{ not small} \end{cases} \quad (3)$$

Here G_F is the Fermi coupling constant, K is a constant, Z is the atomic number, λ is the decay constant, L is the degree of “forbiddenness” of the decay, and α is the fine structure constant. Notice

that the degree of “forbiddenness” appears in an exponent, so that highly forbidden decays are very sensitive to the values of the decay energies Δ , particularly when Δ is small. The decay energies Δ are in turn sensitive to the strength of the nuclear force, particularly for the small values of Δ typical of forbidden decays.

Nuclear Phases

A *phase* is any homogeneous part of a material system separated from other parts by definite physical boundaries, as liquid water and water vapor in a balloon, with water vapor above the liquid. Here “Homogeneous” does not mean only the choices solid, liquid or gas. There are more possibilities than that. In an introductory physics course, along with the concepts of temperature and heat, one learns about the latent heat associated with a first-order phase transition, such as changing liquid water to steam, where 540 calories are required for each gram. Examples of phase transitions include changes in the lattice structure of a crystalline solid, change from normal liquid to liquid crystal, change from superconducting to nonsuperconducting, change from ferromagnetic to nonferromagnetic, superfluid to nonsuperfluid, etc. When we study nuclear matter, one discovers that a rich variety of phase have been proposed and studied (Bonasera, 1999; Schewe, Riordan, & Stein, 2002; Shlomo & Kolomietz, 2005; Snover, Stephens, & Alhassid, 1988; Stephens, 1986).

One type of phase transition is called a shape transition. Nuclei can be stable not only in oblate spheroidal shapes but also as prolate spheroidal shapes (Figure 1).

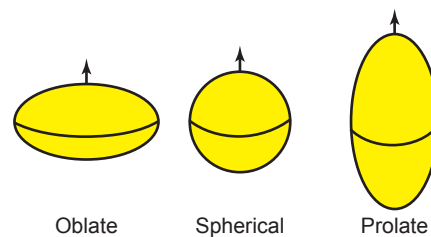


Figure 1. Nuclear shapes. The arrows show the symmetry axis for these axially-symmetric spheroidal shapes.

Rotating liquid drops can also have stable triaxial shapes as was discovered by the French mathematicians Poincaré (1885) and Cartan (1922, 1928). This means that the three principal axes of the shape are all different lengths. Nuclei have more complex possibilities than liquid drops, and under certain conditions can assume these triaxial shapes as well. Figure 2 shows a phase diagram for Osmium-188 according to Goodman (1995a).

In hot (energetic) nuclei, recent data show a change from a phase analogous to a liquid to a gaseous-

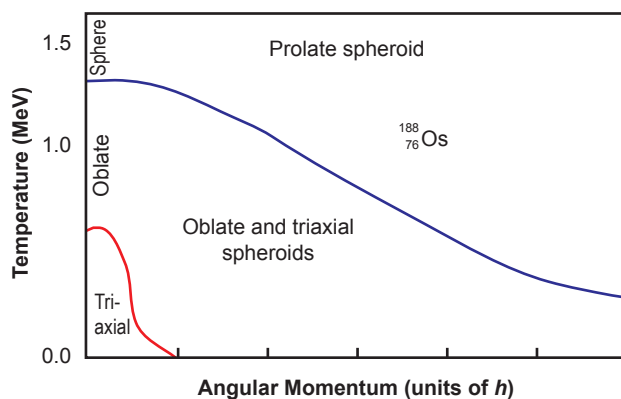


Figure 2. Nuclei have a temperature which can be measured in energy units of MeV, using Boltzmann's constant as the conversion factor. High temperature (energetic) nuclei undergo phase transitions manifested as shape changes. Changes in angular momentum (more rapid rotations) lead to similar transitions, hence we can draw phase diagrams such as the one shown (after Goodman, 1995a).

type phase (Borderie, 2002; D'Agostino et al., 2000; Gulminelli, Chomaz, Raduta, & Raduta, 2003; Lopez, Lacroix, & Vient, 2005; Schewe, Riordan, & Stein, 2002; Shlomo & Kolomietz, 2005; Rivet et al., 2002). In 1973, Brink and Castro showed that when the nucleon density is about one third of the central density of nuclei, a gas of nucleons condenses into α -particles. Thus, near the nuclear surface the density is less, explaining why clusters of α -particles are useful models for many nuclei (Buck & Merchant, 1989; Clark & Johnson, 1978; Delion & Sandulescu, 2002; Hodgson, 2002; Lovas, Kruppa, Beck, & Dickmann, 1987; Tomoda & Arima, 1978; Tonozuka & Arima, 1979). Models such as the *Interacting Boson Model* (Rowe, 2004) lead to a rich phase structure involving three or more control parameters (Bouldjedri & Benabderrahmane, 2003; Caprio & Iachello, 2005; Dieperink & Scholten, 1980; Dieperink, Scholten, & Iachello, 1980; Feng, Gilmore, & Deans, 1981; Scharff-Goldhaber, 1980). In this model pairs of protons or neutrons are formed, and the nuclear properties must be explained taking this into account. Three different symmetries emerge naturally from this model, labeled U(5), SU(3) and SO(6) according to the groups needed to label the nuclear states. They correspond to spherical nuclei, ellipsoidally deformed nuclei with axial symmetry and soft triaxial nuclei, respectively. Empirical manifestations of these structures have been found throughout the nuclear chart and are now seen to represent the commonly occurring shapes that the nucleus adopts. To discuss this intelligently, one needs to review the theory of superconductivity, and consider how it applies to nuclei, which we will do in the next section.

Cooper Pairs and the Mechanism of Superconductivity

Bardeen, Cooper, and Schrieffer (1957) (hereafter referred to as BCS) proposed that a very weak attractive force between pairs of electrons with opposite momentum was responsible for superconducting currents observed in many materials at low temperature. This at first appears counterintuitive since electrons are negatively charged, and two like charges should repel each other. However, the BCS theory is not concerned with two electrons moving in a vacuum, but in a crystalline solid where the medium includes a regular arrangement of positive ions. We picture the solid as a three-dimensional arrangement of positive ions called the *lattice*, surrounded by a gas of free electrons. The idea is that one electron attracts the positive ions near to it, causing a distortion of the crystal lattice, which may be transmitted through the solid in the form of quantized elastic waves, called *phonons*.

As the electron moves through the lattice, the positive ions nearby are displaced, forming a thin tube of displacement which follows the electron as it moves through the lattice. A second electron may be attracted to the tube of concentrated positive charge, but the attraction is only large if an electron moves along the direction of the tube opposite to the direction of the first electron. Otherwise the encounter is too short and the attractive interaction is too weak. Also, this mechanism is only effective at low temperature, where other lattice motions do not interfere. The two electrons, together with their tubes of displaced ions trailing along behind them, form a quantum state called a *Cooper pair* or also a *quasiparticle*.

The electrons in a crystal obey the *Pauli exclusion principle*, which states that no two electrons can occupy the same state unless they have opposite spins. Having the opposite spin also makes the state different, so taking spin into account enables the statement to be changed to the requirement that no two electrons can occupy the same state. Particles which obey the Pauli exclusion principle have half-integral spin and are called *fermions*. The other category of particles includes particles of integral spin, called *bosons*, and these particles do not obey the Pauli exclusion principle and can have more than one particle per quantum state. In a solid, there is an amount of energy called the *Fermi energy*, named after Enrico Fermi, which at absolute zero would divide occupied electron energy states from unoccupied energies. At a finite temperature, the Fermi energy still divides occupied levels from unoccupied levels, but the boundary is not so sharp, the occupation numbers changing from one to zero only over a finite interval or spread of energy. This means that Cooper pairs can only be formed by electrons near to the Fermi energy, because the energy

from the weak attractions of one electron for another can only be effective if the electrons can change their quantum states to unoccupied levels.

The attractive force between two electrons is also only effective for two electrons which approach nearly head on, since otherwise their distances of approach are too large for this weak mechanism to work. By “head-on” we mean that the two opposite trajectories are along the same line. Also, the Pauli principle requires the two electrons to have opposite spins. Thus the Cooper pair can be visualized as a head-on back and forth movement of the two electrons, sort of like performers Fred Astaire and Ginger Rogers dancing back and forth on a stage. However, in the case of the Cooper pair, as time goes on all directions in the crystal are covered with equal probabilities as the two electrons separate from the point of closest approach. This may be visualized like the quills of a porcupine pointing in all directions from the center (Figure 3). In Figure 3, the lines extend out from the center a distance d of the order of the coherence length, which is about 10,000 lattice diameters (Weisskopf, 1979). Weisskopf, in the preprint cited, showed that when an external electric field is applied the Cooper pairs enable a current to exist which is not hindered by the mass of the electron. Hence there no electrical resistance develops. However, that is not the subject here.

This mechanism for the BCS theory involves an attraction caused by the distortion of the crystal lattice. Quantized elastic waves in a solid are called phonons, but the phonons involved here are called *virtual* phonons, since they exist only over the short

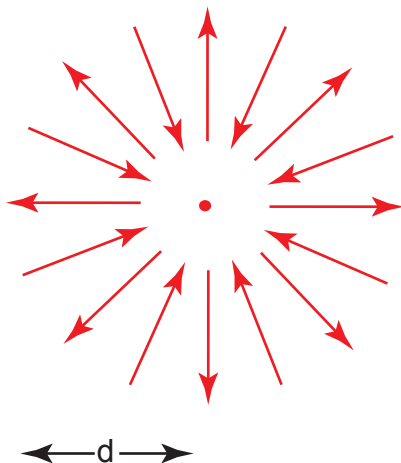


Figure 3. After Weissopf, 1979. Pairs of electrons with opposite momentum are attracted because of their interaction with the positive ions of the crystal. The electrons of a pair approach each other along linear trajectories with zero angular momentum but they scatter in other directions at the center, leading to a spherically symmetric distribution over time. The distance d is the coherence length, which is about 10,000 lattice spacing distances.

distances of interaction of the two electrons of the Cooper pair. Phonons which escape to infinity would be called *real* phonons. This BCS mechanism is not thought to be responsible for the pairing that occurs in the case of the new high-temperature superconductors, such as the so-called “one-two-three” material $Y_1Ba_2Cu_3O_{7-x}$. It has been suggested that quantized spin waves in the solid replace the phonons in this case, but the subject is still somewhat unsettled.

In the case of the nucleus, it was suggested early on that pairs of protons or pairs of neutrons might exist in the nucleus, forming quasiparticles whose existence changes the properties and energy levels of the nucleus (Belyaev, 1959; Griffin, 1963; Kerman, Lawson, & MacFarlane, 1961). Mottelson and Valatin (1960) suggested that rapid rotation of the nucleus might break up the quasiparticles just as a strong external magnetic field will break up the Cooper pairs in the crystalline solid.

Pairing in Nuclei

As we have seen, the theory of superconductivity led to progress in the understanding of pairing phenomena due to its concept of Cooper pairs. Another line of research concerned superfluid helium. At extremely low temperatures, and ordinary pressures, helium refused to form a solid but instead formed a liquid which could flow without viscosity. For some time the close similarity between superconducting metals and the properties of superfluid helium was not understood. It was known that helium owed its superfluid properties to the boson nature of the ^4He atom, with its total spin of zero. ^4He atoms are *bosons*, and as such these particles do not obey the Pauli exclusion principle and can have more than one particle per quantum state. However, the electrons in a metal are *fermions*! That is why the pairing concept introduced by Cooper (1956) was so important. Cooper showed that two fermions of opposite spin attract each other to form a bound state, and that this *Cooper pair* has zero spin and behaves like a boson. This idea led to the basis of the BCS theory of superconductivity of Bardeen, Cooper, and Schrieffer (1957). The rudiments of the pairing idea had been around since Eugene Wigner’s (1937, 1939) applications of symmetry mathematics to nuclei but the full-fledged adoption of BCS theory into nuclear physics followed the tentative plans offered by Bohr, Mottelson, and Pines (1958) and the more detailed form given by Belyaev (1959). Bohr et al. explained the energy gap observed in the spectra of even-even nuclei in terms of the BCS ideas, and then Belyaev used the mathematics of field theory, and approximations that followed from it that made possible simple calculations of the effects of pairing in nuclei in terms of independent quasi-particles. Pairing theory was then adopted in nuclear physics as

a fundamental concept for describing binding energies of nuclei and their low-lying vibrational spectra. Nuclear superconductors (or superfluids) have an energy gap in their spectrum, but there are also collective excitations corresponding to rotational or vibrational states. Unlike superconductors, however, nuclear excited states typically last only nanoseconds to picoseconds, since they can decay by emitting gamma-radiation.

In nuclei pairs of neutrons and also pairs of protons couple to a total spin of 0, giving a little extra binding energy. This has been noticed in several lines of research. Experimentally we find that nuclei with an even number of protons and also an even number of neutrons have a total angular momentum of zero in their ground states. The nuclear force just naturally leads to this. It is thought to be due to the short range portion of the interaction between nucleons. The ground states of the majority of nuclei are very well described in terms of “superfluid condensates,” in which pairs of protons or pairs of neutrons form. Most nuclear ground states are very well described in terms of a lowering of the total nuclear energy by the joining of pairs of nucleons into pairs, just like the Cooper pairs of electrons in superconductors. Experimentally, when charts and tables of nuclear binding energies, or nuclear masses are examined, we find a systematic lowering of the nuclear mass when we compare odd to even mass numbers. Mass formulas constructed to fit the nuclei need a term which is negative for even mass number and even proton number, but positive for even mass number and odd proton number. The even-even nucleus is more bound than the even-odd nucleus.

Although the pairing force is very schematic, it nevertheless explains many systematic features of nuclear spectroscopy. Even Z -even N nuclei invariably have spin-zero ground states and very few low-lying excited states. Odd-mass nuclei, for which the low-lying states are one quasiparticle states, are more complex; and this explains why odd Z -odd N nuclei, for which low-lying states are a proton quasi-particle coupled to a neutron quasi-particle, are still more complex. The odd-even mass difference is also explained: a single quasi-particle has a *minimum* energy equal to the “pairing gap” Δ . Kisslinger and Sorensen (1963) defined three odd-even mass differences

$$P_p(Z, N) = E(Z-1, N) + E(Z+1, N) - 2E(Z, N) \quad (4)$$

$$P_p(Z, N) = E(Z, N-1) + E(Z, N+1) - 2E(Z, N) \quad (5)$$

$$P_{np}(Z, N) = E(Z+1, N-1) + E(Z-1, N+1) - 2E(Z, N) \quad (6)$$

where in equation (4) A is odd, N even; in (5) Z is even, N odd; and in (6) both N and Z are odd integers.

$E(Z, N)$ is the binding energy of the Z, N nucleus. Aside from the effect of the long-rang part of the force, which we ignore, these mass differences are simply related to the quasi-particle energies. P_p compares an odd- Z nucleus to the adjacent even-even nuclei and should thus just be equal to $2E_p$, twice the energy of the ground-state proton quasi-particle. Similarly, $P_n = 2E_n$ and $P_{np} = 2E_n + 2E_p$ where E represents in each case the ground-state quasi-particle energy.

The last unpaired particle in an odd-mass nucleus is much less strongly bound than in the neighboring even nuclei. Assuming that the ground state of the odd-mass nucleus is a single quasiparticle state j , we have

$$E_j = \sqrt{(\varepsilon_j - \varepsilon_F)^2 + \Delta^2} \geq \Delta$$

Here ε_j is the energy level of the last nucleon (perhaps determined from the shell model), ε_F is the Fermi energy, Δ is the pairing gap, and E_j is the quasiparticle energy. Recall that the Fermi energy is a number which marks the approximate division of occupied levels from unoccupied levels. Thus a calculation of $P_p(Z, N)$ using experimentally determined masses can be used to estimate E_j , and hence Δ . Mass and binding energy data agree well with this concept, and it is used to estimate the value of the pairing gap Δ . However, as Kisslinger and Sorensen noted the overall accuracy to which parameters such as this are known is not more than about 20% for most isotopes (Kisslinger & Sorensen 1963, p.862). A study of the literature shows that the theory continues to be plagued by these inaccuracies (Bai & Hu, 1997; Launey, 2003).

Above a certain critical temperature, conventionally measured in MeV in nuclear physics using Boltzmann's constant as the conversion factor, there is found the “pairing phase transition.” Calculations give a critical temperature of 0.5 to 1.0 MeV, the exact value depending on the particular nucleus. However, the analogy to a large system with an Avogadro's number of particles is not perfect, because nuclei have only a finite number of particles. Hence, the energy for the loss of pairing is not a sharp value, but is spread over a finite range (Goodman, 1981, 1983; Moretto, 1972a, b).

Theoretical Calculation of the Pairing Gap Δ

In nuclear models such as the shell model (Haxel, Jensen, & Seuss, 1949; Mayer, 1949, 1950), the neutrons and protons are imagined to move in a centrally-symmetric potential. However, we know that not all of the two-body interaction is adequately represented by such an approximation. Rowe (1970) considered a multipole expansion of the exact interaction. From this he demonstrated the tendency of short-range components of the forces is to couple

particles pairwise to stable configurations with zero total angular momentum. This helps to explain the origin of pairing forces but does not provide a very precise way of calculating their strengths.

Belyaev (1959) considered the similarity between the pairing energy of two nucleons in a nucleus with opposite projections of angular momentum and quasiboundstates of electron pairs in a superconductor with equal and opposite momenta. Belyaev began by using the shell model for an initial estimate of the energy levels ε_j , where $j=1, 2, \dots, Z$ (or N) labels the protons (or neutrons if we are considering neutron pairing) in the nucleus under study. He introduced an unknown pairing interaction strength parameter G . He further considered an approximation for which certain quantities, called matrix elements, were the same between any levels inside a shell. From this he derived two coupled equations which must be solved for the energy gap Δ , and Fermi energy ε_F of this nucleus. The two equations are the Gap Equation:

$$\frac{G}{2} \sum_{v>0} \frac{1}{\sqrt{(\varepsilon_v - \varepsilon_F)^2 + \Delta^2}} = 1 \quad (7)$$

and the number constraint equation (which requires the total number of particles to equal the actual number):

$$\sum_{v>0} \left\{ 1 - \frac{\varepsilon_v - \varepsilon_F}{\sqrt{(\varepsilon_v - \varepsilon_F)^2 + \Delta^2}} \right\} = N \quad (8)$$

Here G is a number called the “pairing strength,” N is the total number of particles, and the other quantities were defined above. A solution to these equations does not necessarily exist. Early in the history of the application of this BCS theory to nuclei (Nilsson, et al., 1969), the overall A -dependence of G was found to be proportional to A^{-1} . Theoretically this corresponds to the fact that in lieu of any other correlations, the so-called overlap integral should be inversely proportional to the volume of the nucleus, and that the radius R of the nucleus is proportional to the cube root of the mass number A so that $(4\pi R^3/3)^{-1}$ is proportional to A^{-1} . Hence, Nilsson et al. gave the equation

$$G \times A = g_0 \pm g_1 \frac{N - Z}{A}, \quad (9)$$

with $g_0=19.2\text{MeV}$ and $g_1=7.4\text{MeV}$. The term proportional g_1 takes into account that the pairing strength is found to change when the number of neutrons (N) does not equal the number of protons (Z). The plus sign in the equation is for protons, the minus sign for neutrons.

Closed shell nuclei such as oxygen-16 ($Z=8$, $N=8$, both closed shells or “magic numbers” in the shell model) have a small density of states near the Fermi level λ , and either have a relatively small pairing gap Δ or no pairing gap at all. In some nuclei there is a competition between deformation and superconductivity, and deformed states exist which are not superconducting ($\Delta=0$). Also, in light nuclei, nuclei with mass numbers less than about 20, the level density may be too small for pairing solutions to exist. For instance, Hagino and Bertsch (2000) presented results for the simple BCS (or Belyaev) model in which no solution existed for oxygen-15, -16, or -17, but did exist for oxygen-14 and oxygen-18. This does not mean that the pairing strength G in the equation above is zero, just that no solution for the pairing gap Δ exists and the spectrum of energy levels will not be modified accordingly. This becomes particularly significant when we consider that near a closed shell configuration, the critical temperature for a phase transition is lowered considerably (Alhassid, Manoyan, & Levit, 1989).

In order to estimate the variation of the pairing gap with change in the strength of the nuclear force, hence with change in the value of G , the pairing strength, an algorithm is needed to generate the single particle energies ε_j and use them to calculate the pairing gap Δ and Fermi energy ε_F . For this purpose, I used some old work (Chaffin, 1973; Chaffin & Swamy, 1972; Chaffin, Dickmann, & Swamy, 1975; Swamy, 1969) to generate the single particle energies ε_j appropriate for the nucleus being considered. The simple procedure for calculating nuclear energy levels is as follows. The value of λ^2 , the oscillator constant for the nucleus, is determined (for example, for ^{208}Pb it is 0.17 fm^{-2}) consistent with the parameter that could reproduce experimental Coulomb energies (Chaffin & Swamy, 1972). With this value we apply the equations:

$$\varepsilon = \sqrt{m_0^2 c^4 + 4\lambda^2 (\hbar c)^2 (v + |\kappa| + \frac{1}{2})} - \frac{4\lambda^2 (\hbar c)^2}{m_0 c^2} \left[|\kappa| + \frac{1}{2} s_{-\kappa} \right] \quad (10)$$

Here m_0 is the effective mass of the nucleon, while \hbar is Planck’s constant over twice π , c is the speed of light, and v , κ , and μ are quantum numbers labeling the single-particle states: v may be called the “shell number,” κ is related to the orbital angular momentum quantum number via $\kappa=\ell$ if $j=\ell-\frac{1}{2}$ or $\kappa=-\ell-1$ if $j=\ell+\frac{1}{2}$, and μ is the projection of the total angular momentum on the z -axis. For the example of lead, Pb-208 , this gives the energies in MeV:

$$\epsilon = 938.256 \sqrt{1 + (0.030)(v + |\kappa| + \frac{1}{2})} - 7.03 \left\{ |\kappa| + \frac{1}{2} S_{-\kappa} \right\} \quad (11)$$

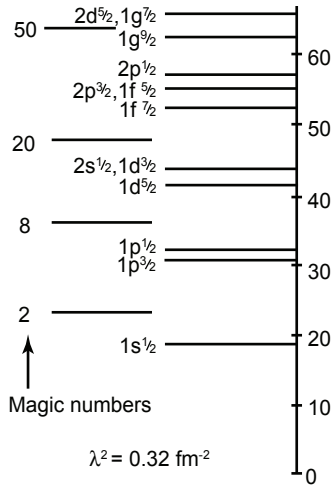


Figure 4. Energy levels calculated with equation 10 for the case of a light nucleus. The scale at the right is in MeV.

Figure 4 shows a plot of the energy levels for a light nucleus obtained in this way.

According to Kaiser, Niksic, and Vretenar (2005) a typical value of the effective mass is about 0.64 times the rest mass of the proton. With this equation, a *Mathematica* algorithm easily generated all the energy levels needed.

Once we have the single-particle energies, we need to solve the gap equation and the number constraint equations (7), (8) given above for Δ and λ . For this purpose, a **Fortran** program originally developed by Dr. Friedrich Dickmann, and modified slightly during later applications (Chaffin & Dickmann, 1976a,b, 1977; Dickmann, 1980) was revived and adopted to the problem. The results will be given in connection with a discussion of individual nuclei of importance in radioisotope dating.

Carbon-14

The decay of ^{14}C into ^{14}N is evidently a transition of the type $i=0 \rightarrow f=1$, no parity change. ^{14}C is an allowed decay, but a Gamow-Teller transition. According to Parrington et al. (1996) the energy release is 0.156475 MeV, no gamma-ray is emitted, and the half-life is 5,715 years. The decay cannot go to the ^{14}N state with the same spin because that ^{14}N state is higher in energy (see Figure 5). So the decay must proceed to a state with a different spin. However, the “matrix-element” or Gamow-Teller factor is unusually

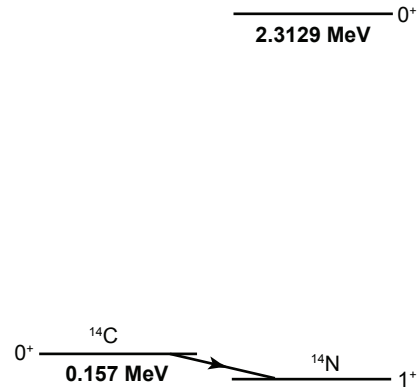


Figure 5. Energy levels, shown to scale, in the ^{14}C decay. Data from Lederer and Shirley (1978). The nearest excited state on ^{14}N is at 2.3129 MeV above the ground state, and the first ^{14}C excited state is even higher. The numerous higher energy levels are now shown.

small in this case. Somewhat successful attempts to explain this have been based on a “tensor” component of the nuclear force (Jancovici & Talmi, 1954; Rose, Häusser, & Warburton, 1968). For ^{14}O , a nucleus that might be thought to be somewhat similar, the β^+ transition is a pure Fermi transition and the matrix element is $\sqrt{2}$, superallowed according to Burcham (1963, p. 607) or Kaplan (1963, pp. 371–373).

For proton pairing strength $G_p=1.5$, the pairing gap was found from the Fortran program mentioned above to be $\Delta=0.464\text{MeV}$ and the Fermi energy $\epsilon_f=41.142\text{MeV}$. For other pairing strengths, the results are shown in Figure 6, which shows a limit of 1.48 MeV below which no pairing gap exists according to the program. According to the Figure 11.3 caption of Rowe (1970), the pairing strength G should be of the order of magnitude between $19/A=1.36\text{MeV}$ and $23/A=1.64\text{MeV}$ for light nuclei such as C-14. Hence $G_p=1.5\text{MeV}$ would be acceptable for this nucleus. However, more recent work (Bai & Hu, 1997) gave a different estimate. They gave $G_n=G_p=0.48$ for carbon. If this is correct then there would be no pairing gap. This could mean that the half-life of ^{14}C is not sensitive to a change in the strength of the nuclear force.

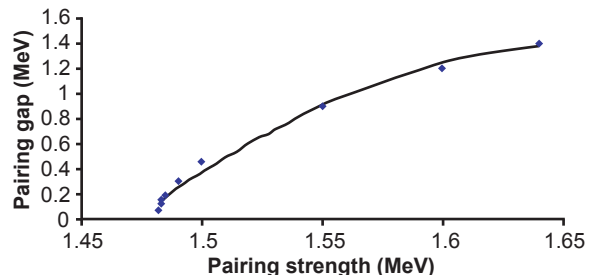


Figure 6. Pairing gap versus pairing strength for ^{14}C . As indicated, no pairing gap exists for ^{14}C for a pairing strength below about 1.48 MeV

In another relatively recent work Åberg, Semmes, and Nazarewicz (1997) gave the formula:

$$G_p = \frac{1}{A} [17.9 + 0.176(N - Z)] \quad (12)$$

and this gives $G_p = 1.30 \text{ MeV}$ for ^{14}C . This too would indicate that no pairing gap exists for ^{14}C .

Potassium-40

As shown in Figure 7, potassium-40 decays 89.3% of the time by β -minus decay to calcium-40, 0.001% of the time by β -plus decay to the ground state of argon-40, 10.51% of the time to the 2^+ excited state of ^{40}Ar (which then very rapidly decays to the ground state by γ -decay) by electron capture, and 0.16% of the time directly to the ground state of ^{40}Ar by electron capture. Note that the β^+ decay of ^{40}K cannot go to the 2^+ state (but can to the 0^+ ground state) because the energy release, called the Q-value) must be big enough to form the rest mass energy of two electrons [1.02 MeV], and this amount of energy would not be available. If, due to a change in the strength of the nuclear force, the 2^+ state were to shift downward by half of 1.02 MeV, or the 4^- state of ^{40}K upward by this amount, or if any combination of relative shifting of these two states totaled 1.02 MeV, then the β^+ decay could contribute also, which could result in accelerated decay. Also, if the 3^- state of ^{40}K shown at 0.0296 MeV were to shift below the 2^+ state of ^{40}Ar , the rate for the decay would also be considerably enhanced.

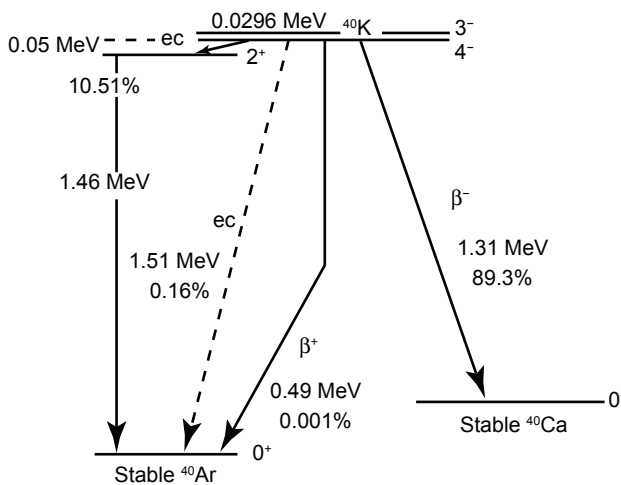


Figure 7. The decay scheme, shown to scale, of ^{40}K also showing the 3^- excited state of ^{40}K . Data from Lederer and Shirley (1978).

According to Rowe (1970, Chapter 11), the pairing strength G should be of the order of magnitude between $19/A = 0.475 \text{ MeV}$ and $23/A = 0.575 \text{ MeV}$ for light nuclei such as ^{40}Ca . Hence the limit point $G = 0.557 \text{ MeV}$ shown on the graph (Figure 8) is acceptable for this nucleus in order for a pairing gap

to exist. However, G_p is barely above the limit where a $\Delta = 0$ result must follow, and a phase transition could occur if the strength of the nuclear force were to change, with an associated decrease in the strength of the pairing strength G . After the phase transition, the nuclear properties could be expected to be much different, with a resulting change in half-life.

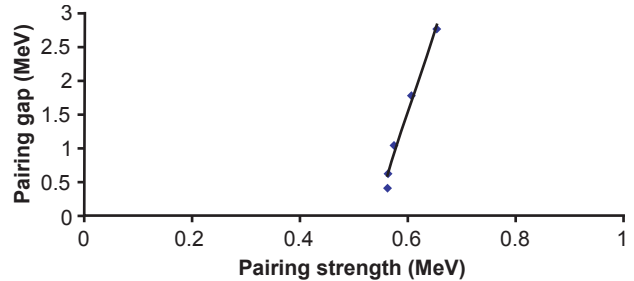


Figure 8. Graph showing the results of the present work of pairing gap versus pairing strength for ^{40}K .

One question that might be asked is whether a nucleus such as ^{40}K would spend an appreciable amount of time in an excited state, and whether the half-lives of those excited states might contribute to the depletion of the ^{40}K and the accumulation of the daughter products. The first excited state of ^{40}K has a half-life for gamma decay of 4.26 nanoseconds (4.26×10^{-9} seconds), and of the next ten excited states listed by Lederer and Shirley (1978, p.92), none has a half-life greater than 1.64 picoseconds (1.64×10^{-12} seconds), with an average of (0.513 ± 0.49) picoseconds, where the plus or minus margin is one standard deviation. At a temperature $T = 293$ Kelvins, kT is 0.025 eV. If the energy ϵ is 1 eV then $\exp[-\epsilon/kT]$, which occurs in a Maxwell-Boltzmann distribution function, is 4×10^{-18} . For higher energies, this factor is even smaller. This means that for low-lying nuclear energy levels, typically spaced by 0.1 MeV or more, which is 10^5 eV , the occupation probability of nuclei in equilibrium with the surrounding radiation is impossibly low. Hence no appreciable depletion of ^{40}K from this source could be expected for a rock that remains anywhere near room temperature.

Rubidium-87

Rubidium-87 decays to Strontium-87 with a positive energy release $Q = 0.283 \text{ MeV}$ so β^- decay is possible (Figure 9). The nuclear spin changes by

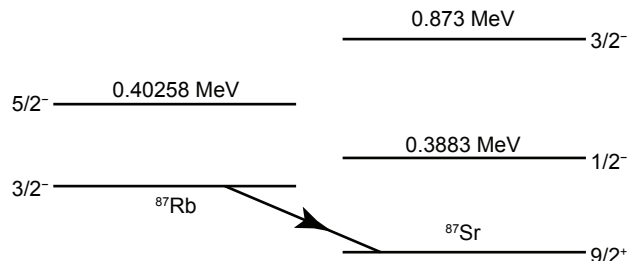


Figure 9. Energy levels for the β -minus decay of ^{87}Rb . Data from Lederer and Shirley (1978).

$\Delta I = 3/2 - 9/2 = -3$ with parity change so this is a forbidden decay, called third forbidden (Sastry, 1969).

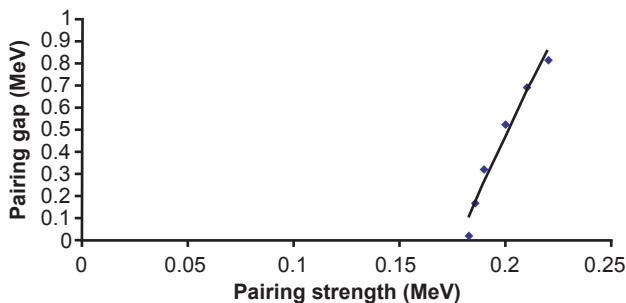


Figure 10. The pairing gap versus pairing strength for ⁸⁷Sr.

The results of the calculation of the pairing gap versus pairing strength are shown in Figure 10.

According to Rowe (1970), the pairing strength G_p for protons should be between 0.218MeV and 0.264MeV in the strontium-87 region of the chart of the nuclides. According to the calculations, the pairing gap should then be between 0.8MeV and 1.59MeV. A reduction of the pairing strength G_p to about 0.18MeV would cause the disappearance of the pairing gap.

Lutetium-176

For Lutetium-176, 99.1% of the decay proceeds to the 6⁺ excited state of Hafnium-176 at 0.597MeV above the ground state (Figure 11). The energy released is 0.57MeV (Parrington et al., 1996). The nuclear spin changes by $\Delta I = 6 - 1877 = -1$ with parity change so this is a first forbidden transition. The other 0.9% proceeds to the 8⁺ excited state at 0.998MeV above the ground state, with $\Delta I = 8 - 7 = +1$ and a parity change. There is a 4⁺ state at 0.2902MeV above the ground state, a 2⁺ state at 0.08835MeV above the ground state, a 0⁺ state exists at 1.15MeV above the ground state and a 2⁺ state at 1.2264MeV, but decay to these states would be highly forbidden due to the high spin of the parent nucleus ¹⁷⁶Lu.

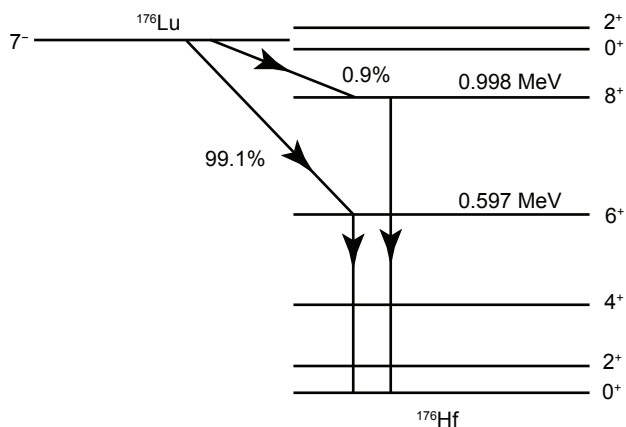


Figure 11. Level scheme for the beta-minus decay of ¹⁷⁶Lu. Data from Lederer and Shirley (1978) and Enghardt, et al. (1999).

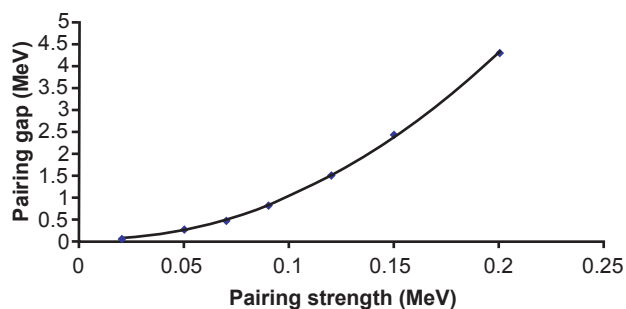


Figure 12. Pairing Gap versus pairing strength for the ground state of ¹⁷⁶Hf.

Figure 12 shows the results of a calculation of the pairing gap for the ground state of ¹⁷⁶Hf. The graph indicates that this state of ¹⁷⁶Hf is not on the brink of phase change, since there is a smooth transition of the graph towards zero as the pairing strength gets smaller. However, as Figure 11 indicates, most of the decay is to the 6⁺ excited state of ¹⁷⁶Hf rather than to the ground state. Hence, these results, which apply to the ground state, may not be accurate for this excited state, and are not very conclusive for this nucleus. For heavy nuclei such as ¹⁷⁶Hf, the energy levels given by our model equation (10) above are not quite in agreement with experiment, so the energy levels near the Fermi level had to be adjusted using experimental data given by Brink & Vautherin (1970) and Bromley & Weneser (1968). The model equations serve to generate energies of the levels and their multiplicities, hence it is easy substitute more exact information when it is available. For instance, the model equations put the 1h_{11/2} level at 41.737MeV, which is above the 2d_{3/2} level whereas experimental spins and parities indicate that it should be below. Hence it was changed to 38.584MeV and other levels near the Fermi energy were changed using the numbers given by Bromley & Weneser.

Rhenium-187

The decay of Rhenium-187 (Figure 13) is a 1st forbidden β^- transition with a large atomic number and very small decay energy 0.0026MeV. The nuclear spin changes by $\Delta I = 1/2 - 5/2 = -2$ and the parity changes. If the decay could proceed to the excited state at 0.0098MeV, it would be an allowed transition, and would be extremely accelerated. This is in fact what has been found experimentally when the electrons are stripped from the ¹⁸⁷Re nucleus (Ashktorab, Jänecke, Becchetti, & Roberts, 1993; Bosch et al., 1996). According to Moller, Nix, and Kratz (1997), the binding energy of the Z electrons comprising an atom, can be approximated by $a_{el}Z^{2.39}$, with $a_{el} = 1.433 \times 10^{-5}$ MeV, hence for $Z = 76$ this gives 0.45MeV, which is more than the separation between the two low-lying levels of ¹⁸⁷Os, which helps to explain the extreme enhancement in decay rate observed by

Bosch et al. (1996) when the electrons were stripped away. However, for a nucleus such as ^{40}K , with $Z=19$, the electronic binding is only 0.0163MeV according to the formula, which is too small to initiate similar effects in a light nucleus such as ^{40}K . For ^{87}Rb , the electronic binding is 0.08MeV, which is near but slightly less than the energy needed to bring the $\frac{1}{2}^-$ excited state down below the level of the $\frac{3}{2}^-$ ground state of the parent nucleus. However, these nuclei are never stripped of all their electrons while in a rock situated near the surface of planet earth, so this is all probably irrelevant unless the problem deals with the interiors of stars. Most of the binding energy of the electrons is from the innermost shells (the 1s shell in particular), and the outer shell electrons have a binding energy of only a few electron volts, which is a few millionths of an MeV. Hence, the removal of a few outer electrons is of no consequence.

For ^{187}Os , as for ^{176}Hf , the energy levels given by our model Equation (10) above are not quite in agreement with experiment, so the energy levels near the Fermi level had to be adjusted using experimental data given by Bromley and Weneser (1968) and Brink and Vautherin (1970). When this was done, the results shown in Figure 14 are obtained. The minimum

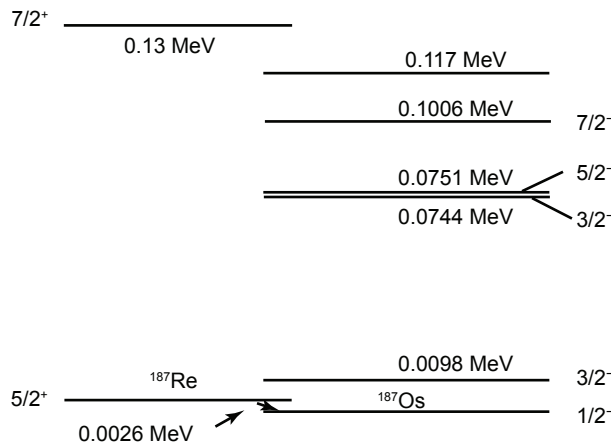


Figure 13. Level scheme for the decay of ^{187}Re . Data are from Lederer and Shirley (1978) and Morgen, Nielsen, Onsgaard, and Sondergaard (1973).

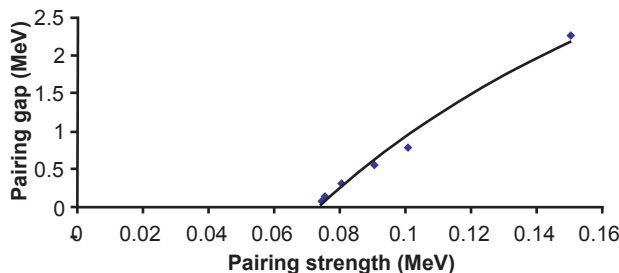


Figure 14. Pairing gap versus pairing strength for ^{187}Os .

pairing strength needed to have a BCS model solution is about 0.07MeV, but the actual pairing strength for $A=187$ according to Rowe (1970) is about 0.1 to 0.12MeV. Hence, the results show that ^{187}Os should have a pairing gap of about 0.78MeV, but that a slight reduction in pairing strength would be sufficient to cause this nucleus to undergo a phase transition to a non-superfluid [or non BCS] state.

According to Sauvage-Letessier, Quentin, and Flocard (1981), the proton pairing strength G is 0.063 to 0.066MeV for osmium isotopes of about this neutron number. They noted that the osmium isotopes have low lying energy levels which can have various shapes, a phenomenon called “shape coexistence.” For ^{188}Os they estimated proton pairing gaps Δ between 0.74 and 1.03MeV for positive quadrupole moments (prolate shapes) and pairing gaps Δ between 1.00 and 1.20MeV for negative quadrupole moments (oblate shapes). Later work by Goodman (1995a, 1995b) has already been mentioned which also considered triaxial deformations. Goodman’s work showed that excitation of a non-rotating ^{188}Os nucleus to a “temperature” of about 1.33MeV would wash out the quantum shell effects and cause this “hot” nucleus to assume a spherical shape (see Figure 2). Goodman did not discuss ^{187}Os , but we might expect the energy needed for a phase transition at the same order of magnitude. From Figure 14 and the values of the pairing strength G given by Sauvage-Letessier, Quentin, and Flocard, it is evident that the ground state of ^{187}Os is poised on the brink of a phase transition from superconducting to nonsuperconducting. This means that a very small change in the strength of the nuclear force could drastically change the properties of this nucleus and hence the half-life for the rhenium-osmium decay.

Energy levels of ^{187}Os were investigated experimentally by Morgen, Nielsen, Onsgaard, and Sondergaard (1973) and by Ahlgren and Daly (1972). Most of the levels are very short lived, but there is an isomeric level at 0.257MeV with a half-life of 231 microseconds. Interestingly, Ahlgren and Daly reported that the level structure of ^{185}W is very similar to that of ^{187}Os . ^{185}W has two less protons than ^{187}Os but the same number of neutrons, 111. However, the $\frac{1}{2}^-$ level is above the $\frac{3}{2}^-$ level in ^{185}W rather than below it and hence is the ground state. This is caused by a shift of a few kiloelectron volts in energy, which is a relatively small shift for nuclei. Thus the rhenium-osmium case illustrates in several ways how small energy changes can be important in decay schemes.

Low Temperature Enhancement of Decay Rates

A recent episode in the literature has been the enhancement of the decay rate of Beryllium-7 at low temperatures (Limata et al., 2006a, b; Wang

et al., 2006). According to Claus Rolfs, a German physicist, electrons in a metal crowd around a positively charged nucleus at low temperature and their attraction alters the decay rates. According to Rolfs (Kettner, Becker, Strieder, & Rolfs, 2006), the decay of ^{210}Po can be enhanced due to this screening energy, which is 0.410 MeV. However, Rolfs uses the Debye screening model to derive the qualitative and quantitative effects he quotes. This Debye model, dating from the 1920s, oversimplifies the situation and there are indications from experiment that the effect is not as large as Rolfs' model would predict. Huh (1999) had measured a change in the ^7Be half-life of 1.5% depending on the chemical environment. Limata et al. (2006a) found no change in the half-life of ^7Be , to within the experimental errors of 0.4%, when a ^7Be beam was implanted in different metallic targets. Wang et al. (2006) implanted ^7Be in metallic targets and also cooled the materials to 12 Kelvin. They found a change in half-life of up to 0.9% for metallic targets and no change for insulating targets. While this change is non-zero, it was not as much as Rolfs' simple model prediction. Limata et al. (2006b) and Spillane et al. (2007) found the change in half-lives for ^{22}Na and ^{198}Au to also be smaller than the considerations based on the Debye-screening model. It seems that the results thus far indicate no change in half-lives of more than about 4%, with the exception of an old report of a 40% decrease in the radioactivity of tritium embedded in small titanium particles (Reifensweiler, 1994). Zinner (2007), on the basis of theoretical calculations using both the Thomas-Fermi and Debye models concluded that "our calculations do not support the exciting idea that nuclear waste can be faster disposed of if embedded in metals at low temperatures due to significantly reduced lifetimes." Although more results will undoubtedly be forthcoming, one may conclude that these alterations in the environment of a nucleus are not likely to produce the large enhancements that have been imagined by Rolfs et al. However, they do indicate that an experimental alteration of the potential energy of the nucleus can lead to measurable effects, and are an encouragement to the present work which involves instead a hypothesized change in the strength of the nuclear force. An additional consideration is of course the obvious fact that earth rocks were never cooled to 12 Kelvins, for any model of earth history yet seriously proposed, during their existence on planet earth.

More General Phase Transitions

Experimental and theoretical studies of the low-lying excited states of nuclei reveal that some low-lying states exist with a spin and parity of 0^+ . They are the lowest lying states of a series of states referred to as a "rotational band" (Rowe, Thiamova, & Wood,

2006). For example, Figure 11 shows the ground state "band" of ^{176}Hf with spins and parities 0^+ , 2^+ , 4^+ , 6^+ , and 8^+ as well as two low-lying 0^+ and 2^+ levels of a new band beginning at about 1 MeV above the ground state. Low-lying states can have a deformed shape, whether it be an oblate spheroid, and prolate spheroid, or a triaxial shape. Excited nuclear states can have quantized units of vibrational energy, called phonons (Krane, 1987). So-called quadrupole vibrations having a single quantum of vibrational energy carry 2 units of angular momentum and even parity. This leads to the first excited state of an even-even 0^+ nucleus most often being a 2^+ state. If the phonon model is applicable, the second excited state has two phonons of excitation, and involves a triplet of possible states 0^+ , 2^+ , 4^+ . However, as the ^{176}Hf spectrum illustrates, for which Figure 11 only shows the low-lying states, more complicated spectra can also arise (0^+ , 2^+ , 4^+ , 6^+ , and 8^+ states followed by a new band), and a complete description requires more than the use of a simple shell model, and calculations will become more unwieldy. According to Iachello and Arima (1987, p.31), the hafnium spectrum fits the SU(3) phase of ellipsoidally deformed nuclei. Phase transitions have been described in which superconducting states transition into deformed and rotating states (Rowe, Bahri, & Wijesundra, 1998). However, because these nuclei contain a relatively small number of particles, the "phase transition" is not sharp as it would be in a statistical mechanical description of a system with a large number of particles and simple forces acting only between pairs of particles (Moretto, 1972a, b). The "transition" is seen in a series of states of increasing energy, with a decreasing predominance of a successful description in terms of a superconducting state and the rise of the success of a description in terms of deformation and rotation.

In terms of the idea of a change in the strength of the nuclear force, one could expect that there may not be a very sharp change from one phase of the nucleus to another, depending on how much of a change in the nuclear force strength would occur. Rowe (2004) expressed surprise that in spite of these expectations, the phases on either side of the phase transition appeared to be distinct and defined. Rowe described a nuclear system that was spherical in shape but developed vibrational excitations as a function of a vibrational parameter α (not to be confused with the α in Equation 3). As α continued to increase, a relatively sharp transition occurred to a deformed rotational shape, which exhibited rotational states as well as the vibrational states.

Conclusions

For various nuclei, we have seen that a phase

transition could occur if the strength of the nuclear force were to change, with an associated decrease in the strength of the pairing strength G . Nuclei such as ^{187}Os are poised on the brink of a phase transition from superconducting to nonsuperconducting, while ^{14}C is not. Evidences of other phases, such as the $\text{SU}(3)$ phase in ^{176}Hf , exist. The atomic nucleus is a complex system and is subject to major changes which can be caused by a relatively small change in the forces acting on the particles.

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