THE LIFETIME AND RENEWAL OF COMETS

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ABSTRACT

A brief review is conducted of some of the more well-known comets detected in the solar system. The measured attrition rates of Halley's and other comets are used to project a rigorous calculation of their anticipated lifetimes. Some backward projections are made to provide estimates of reasonable upper limits on their current ages.

Some postulated theories for the replenishment of the comet inventory from the assumed "Oort Cloud" are assessed together with proposed schemes for solar capture of their orbits. Orbital mechanics is used to evaluate the probability of the validity of these theories.

INTRODUCTION

In addition to the sun and the planets our solar system contains many other objects such as asteroids and comets. Comets have the property of orbit elongation such that they make periodic approaches to the sun, during which the loss of material to solar pressure leaves one or more tails to impress and awe mankind. Some of these orbits are so elongated (eccentric) as to make their return a thing only to be predicted on the basis of a single solar passage. Others have returned repeatedly and established a schedule so rigorous as to be very beneficial in the development of the science of orbital mechanics.

Comets are normally divided into two categories according to period. The division between short and long periods is somewhere between 100 and 200 years(1). The short period comets are the ones found most interesting since their returns have been observed in modern times and can usually be predicted precisely. Furthermore, observation can reveal much more information about the material properties of the body.

It has been found that a comet cannot be treated simply as a particle in the solution of the two-body problem. Several factors contribute from the variation from an exact elliptical orbit about the sun. First the interaction with other relatively large planets produces perturbations on the standard orbit. Then the comet itself has variations in configuration (not perfectly spherical), mass density, and behavior, since it appears that some of them are spinning and/or ejecting matter in different directions(2). Nevertheless it can be shown that the two-body solution can predict orbits within a very close tolerance, enough to satisfy the purpose of this paper, which is to assess their expected lifetimes.

Most of our specific information about comets comes from the study of the famous Halley's comet. From it we deduce that this and others are composed primarily of frozen water and gases, such as methane and ammonia. The latest measurements indicate that the Halley mass is presently about (10)11 tons and that its mass loss at or near perihelion is about 20 tons per second(3). Extrapolation of these figures and application to the orbits of other short period comets is used to give a predicted lifetime for these objects.

TWO BODY PROBLEM

Six parameters are required to provide a complete description of the solar orbit of a body(4). Three of these numbers describe the orientation and location of the orbit with respect to the plane of the ecliptic and are unnecessary for this study. The other three factors are normally expressed as the period, P, eccentricity, e, and semi-major axis, a. See Figure 1. From this one develops that the perihelion is

$$q = a(1-e) \tag{1}$$

Assessment of a comet's mass loss requires a knowledge of its distance from the sun (the focus of the orbit), since mass loss rate is considered inverse to the squared distance. This is obtained by relating the eccentric anomaly, E, to the mean anomaly, M (Figure 2). The mean anomaly is simply the position on the enclosing circular orbit and is some given fraction of the total orbit expressed in radians. i.e. $P=2\pi$. This relationship is given by the well-known Kepler equation:

$$E - e \sin E = M \tag{2}$$

This nonlinear equation cannot be solved directly. Various approximation and iteration schemes have been suggested. Here we use the method of successive approximations:

$$E_0 = M$$

$$E_1 = M - e \sin E_0$$

$$E_2 = m - e \sin E_1$$

etc.

Having obtained a value for E, we say finally that the distance is

$$r = a (1 - e \cos E) \tag{3}$$

MASS LOSS

The computation of mass loss is based on the assumption that this loss is caused primarily by solar pressure. A postulate that the energy so provided does not attenuate with distance leads to the conclusion that this pressure varies inversely with the square of the distance from the sun. Thus,

$$p = k r^{-2} \tag{4}$$

If the mass loss rate is proportional to pressure then

$$\frac{dm}{dt} = c p \tag{5}$$

so that

$$dm = c p dt$$

$$dm = k r^{-2} dt \tag{6}$$

The maximum mass loss rate occurs at the point of minimum distance, or perihelion. Here M = E = 0 and

$$r = a(1-e) \tag{7}$$

Therefore

$$k = a^2 (1-e) \frac{dm}{dt}\text{max} \tag{8}$$

The mass loss in a single orbit is

$$\Delta m = \int_0^T \frac{k}{r^2} dt \tag{9}$$

Now since $M = \theta = nt$, we have $d\theta = n\,dt$. Substitute to get

$$\Delta m = \frac{2k}{n}\int_0^{\pi}\frac{d\theta}{r^2} \tag{10}$$

where symmetry has been used to consider only half an orbit. With numerical quadrature we get, finally

$$\Delta m = \frac{2k}{n}\sum\frac{\Delta\theta}{r_{av}^2} \tag{11}$$

NUMERICAL SOLUTION

Recent measurements on Halley's comet lead to the assertion that its mass loss rate in the vicinity of perihelion was about 20 tons/second. This is assumed to be the maximum magnitude of dm/dt and is used in the equation above for the computation of k. In the absence of much other meaningful data this value is used for all short period comets to be considered here. The average radius in the summation above is the arithmetic mean of the beginning and ending values in a given segment of orbit.

A Fortran77 program was written to compute the mass loss for a single orbit based on that summation. The first assessment was made with respect to Halley's comet. The first segment in the computation starts at the perihelion. Different numbers of segments were considered, with the number increased until the point was reached at which no significant change in the computed result was realized. This is known as convergence of the numerical solution. In this case a converged result was with 10,000 segments. This does not mean, however, that 10,000 steps are required in the solution. In actuality increases in segment size are made when the incremental ratio $\Delta m/m$ falls below a certain tolerance and an absolute tolerance is set at which all calculation is terminated since further increments of mass loss are trivial at such large distances from the sun.

For a single orbit a certain mass loss is computed. Indications are that the typical comet size is 0.5 to 2 km. With a specific gravity of 1 to 2 (composition is indicated to be frozen water, methane, or ammonia) one deduces a mass of about (10)11 tons, the accepted value of the nucleus of Halley's comet. This value is used for all short period comets to be considered. The single orbit mass loss is divided into the assumed initial mass to yield an estimated lifetime in number of orbits and, therefore a life in years. The program was developed on an Apollo DN3500 engineering work station. Actual computations were completed on a Cray X/MP computer.

Predicted lifetime of Halley's comet is 214 orbits or 16272 years. This compares favorably with the projection given by F. Whipple(5). A complete listing of comets considered(6) is given in Table 1. Note the conclusion that all short comets will be extinct in some 20,000 years and nearly all will have disappeared in about 3,000 years.

COMET REPLENISHMENT

Since the maximum predicted lifetime of the current solar system inventory of comets is limited to 20,000 years some interesting conclusions can be drawn. First, if one assumes that the average comet has lost half of its mass since appearing as a short period comet one can say that the solar system as we know it is only 20,000 years old. An assumption that 5/6 of the average mass has already been lost still leads to a limit of 100,000 years. This becomes additionally significant in view of the fact that assertions are often made that comets contain some of the original material of the solar system. Clearly this conclusion is untenable to most modern astronomers.

The primary explanation advanced to deal with the obvious "youth" of the current comets is that they are continuously being replenished from the "Oort cloud", a collection of millions of similar objects orbiting at a radius of 0.5 to 2 light years, or halfway to the nearest star. So large a number of masses spread out over so large a volume of space is subject to the occasional near miss of some other object (a passing star?). An approach of this type propels the mass into an orbit with a relatively small perihelion, thus making it a long period comet. Now some of these comets are, in turn, transformed into short period comets by passage close to some heavy mass such as Jupiter or Saturn. Hence it is usually argued that the evident low age of the present system of short period comets is no argument against conventional astronomical dating of the solar system at 4-6 billion years. In the discussion to follow an assessment will be made of the mathematical requirements to produce a short period comet in two stages from the hypothetical Oort cloud.

LONG PERIOD COMETS

Assume the radius of the postulated Oort cloud to be one-half year or 34000 AU (astronomical units). The optimum disturbance to a circular orbit of this size would yield a new orbit with aphelion of 34000 AU and perihelion of, say 5 AU, the orbital radius of Jupiter. Clearly the semi-major axis, being the average of the two, is 17000 AU. It should be noted that this would in itself be a very restricted case, since the vast majority of perturbation effects would produce a radial as well as tangential component of velocity change. Whether inward or outward would be inconsequential, since the effect would be to elongate the orbit beyond the postulated size.

Note also that

$$n = (\mu/a^3)^{1/2}$$

The period $P = 2\pi/n$. For a solar orbit $m = 4\pi^2$ and $P = a^{3/2}$. For the given value of a this gives a period of 2.2 million years. Recall that this is a very conservative minimum value.

SHORT PERIOD TRANSITION

A short period comet has a much smaller size orbit than one produced by ejection from an Oort cloud. Note from Eq. (7) that

$$a = q \,/\, (1 - e) \tag{12}$$

where q is the radius of perihelion. A brief review of the results in Table 1 leads to the observation that the maximum a of this collection of comets is 28.9. Let us say conservatively that a short period comet must have a semi-major axis no greater than 100. The investigation is made to determine how this value can be achieved from a starting point of 17000.

The basis of any transition is that a relatively large third body exerts a force such that close passage can significantly alter the orbital parameters of a long period comet. It was demonstrated a century ago by Tisserand(7) that within a nearly spherical volume of space about a third body that body has an influence greater than that of the primary body, even in a heliocentric system. The resulting expression is

$$r_a/r_p = (m/M)^{2/5} \tag{13}$$

The mass ratio of Jupiter to the sun is .00095. With Jupiter's orbit at 5.2 AU we have that

$$r_a = (.00095)^{2/5} \tag{14}$$

$$= .32 \ AU$$

This means that a passage within .32 AU of Jupiter will produce effects large enough to modify perceptibly the orbital parameters of the comet.

Roy(6) presents the equations of orbital perturbation in a three- body system. For our purposes we have

$$\frac{da}{dt} = \frac{2}{n\sqrt{1-e^2}}\left[S_e + \frac{p}{r}T\right] \tag{15}$$

$$\frac{de}{dt} = \frac{\sqrt{1-e^2}}{na}\left[S + T\cos E\right] \tag{16}$$

where S and T are the radial (with respect to the disturbing body, in this case Jupiter) and tangential (in the plane of the comet's orbit) components of the disturbing acceleration experienced by the comet. Also $p = a\,(1-e^2)$.

Now a body passing by a third body would experience the maximum effect when this passage occurs at a right angle to the principal axis of the orbit (Figure 3). In this configuration p = r. The disturbing acceleration is

$$F = -G\, m_1/r = -\mu/r \tag{17}$$

the negative sign indicating that the effect is directed inward.

The minimum distance between the comet orbit and the third body is called d. Then from Figure 4 follows that

$$S = -\frac{\mu}{r}\frac{d}{r};\; T = \frac{\mu}{r}\frac{\sqrt{r^2-d^2}}{r} \tag{18}$$

Since

$$\frac{d}{r} = \cos\alpha;\; r = \frac{d}{\cos\alpha} \tag{19}$$

$$\frac{\sqrt{r^2-d^2}}{r} = \sin\alpha$$

then

$$S = -\mu\frac{\cos^2\alpha}{d};\; T = \mu\frac{\sin\alpha\cos\alpha}{d} \tag{20}$$

Also a segment of orbit, considered rectilinear in this relatively small region, is

$$dx = vdt = rd\alpha,$$

or

$$dt = rd\alpha\,/\,v. \tag{21}$$

If v is assumed constant during passage through the sphere of influence one obtains, finally

$$da = \frac{2\mu}{vn\sqrt{1-e^2}}\left(-e\cos\alpha + \sin\alpha\right)d\alpha \tag{22}$$

$$de = \mu \frac{\sqrt{1-e^2}}{nav} (-\cos\alpha + \sin\alpha \cos E)\, d\alpha \qquad (23)$$

For the earth-sun system

$$\mu = G(M + m_1) = 4\pi^2 \frac{a_1^3}{T_1^2} \qquad (24)$$

$$= 4\pi^2$$

Therefore, approximately,

$$GM = 4\pi^2 \qquad (25)$$

For a planet we have

$$\mu = GM_1 = 4\pi^2 (m/M) \qquad (26)$$

In the case of Jupiter

$$\mu = 4\pi^2 (.00095) \qquad (27)$$

$$= .0375$$

Energy conservation gives the velocity in a solar orbit as

$$v^2 = \mu [(2/v) - (1/a)] \qquad (28)$$

At Jupiter's radius, r = 5.2. With $\mu = 4\pi^2$ we get

$$v = 3.9\ AU/yr \qquad (29)$$

The largest effect occurs when Jupiter is oriented so that $f = \pi/2$. solution for E is gained from,

$$\tan\frac{f}{2} = \left[\frac{1+e}{1-e}\right]^{1/2} \tan\frac{E}{2} \qquad (30)$$

Also

$$r = \frac{a(1-e^2)}{1 + e\cos f} \qquad (31)$$

When $f = \pi/2$, then

$$r = a(1 - e^2)$$

With r = 5.2, we obtain

$$e = .99985. \qquad (32)$$

Solving eqn.(30) gives E = .01782 and

$$\cos E = .99985. \qquad (33)$$

Integration of eq. (22) yields

$$\Delta a = \frac{2\mu_1}{nv\sqrt{1-e^2}} (-2e \sin\alpha_1) \quad (34)$$

$$= \frac{-4\mu_1 e}{nv\sqrt{1-e^2}} \frac{\sqrt{R^2-d^2}}{R}$$

Now d indicates the closest passage distance. In the limit (though clearly impossible) d = 0 and

$$\Delta a = 770{,}000.$$

Likewise eq (23) is integrated to get

$$\Delta e = \frac{\sqrt{1-e^2}\ \mu_1}{nav} (-2 \sin\alpha_1) \quad (35)$$

The limiting value is e = .07.

These integrations were based on the assumption that a and e were modified trivially on their passage through the sphere of influence. Obviously this is not the case and a more complex integration process is required for a rigorous answer. The point of this study is demonstrate that the parameters can be modified sufficiently by close approach to a massive body (in this case Jupiter) to modify the orbit from long period to short period.

TRANSITION PROBABILITY

Having demonstrated that the conversion of a long period comet to short period is theoretically possible it remains to be seen just how likely such a phenomenon would be. Consider the geometrical intersection of the allowable volumes of a comet orbit and the orbit of Jupiter. Since the Oort cloud is assumed spherical a long period orbit can have any inclination and the potential space affected would be an annular spherical shell while the corresponding volume swept out by the planet would be a torus. Both the annular thickness and the toroidal diameter would be the diameter of the sphere of influence. The major radius of the torus would be the radius of Jupiter's orbit.

If we denote the orbital radius of the planet as r1 and the radius of influence as r2 then the annular volume is

$$V_c = (4/3)\ \pi\ [(r_1 + r_2)^3 - (r_1 - r_2)^3] \quad (36)$$

while the toroidal volume is,

$$V_j = 2\pi^2\ r_1\ r_2^2 \quad (37)$$

Now the ratio of V_j / V_c would be that fraction of the comet's trajectory likely to be in the affected zone produced by Jupiter. However this alone would be inadequate to produce the effect in question, since the planet would not necessarily be in the vicinity of the comet when it comes through the torus. Rather it is required to find the intersection of the sphere of influence

$$V_s = (4/3)\ \pi\ r_2^3$$

with the annular volume.

Therefore the probability of being affected is

$$\frac{V_s}{V_c} = \frac{r_2^3}{\left[(r_1 - r_2)^3 - (r_1 - r_2)^3\right]} = \frac{r_2^2}{2\,(2r_1^2 + r_2^2)} \qquad (38)$$

$$= .000945$$

This indicates a one in 1000 likelihood of a comet passing within the distance of a planet adequate to produce the change to short period. With a period previously developed to be 2.2 million years this means that on the average two billion years would be required for an individual comet. It is conceded that perhaps only a minute fraction of long period comets would need to experience this transition to replenish the current supply of short period comets. Nevertheless it should be noted that the time available for a complete turnover is only on the order of 100,000 years based on presently observed data. Furthermore many conservatisms were used in these calculations.

CONCLUSIONS

The preseetly observred hort period comett are being dissipateddaa a relatively rapid rate. All of them will vanish within 100,000 years. A backward extrapolation brings the conclusion that those with which we are presently have been in their present orbits for no longer than the same magnitude of time period - 100,000 years.

A close passage of a long period comet by a massive planetary body - most probably Jupiter - can produce changes in orbital characteristics enough to convert the comet into a short period comet. Note that on the other hand the axis a could be increased by passing on the inside rather than the outside of the third body's orbit.

The probability of a flyby close enough to affect the comet's orbit significantly is no more than one in 1000. Without mention of the many conservative assumptions made this seems insufficient to produce the current supply of short period comets.

There has never been any but the most tenuous of deductions - no evidence - that the Oort cloud actually exists. It is a necessary postulate for the widely accepted long age (about two billion years) of the solar system.

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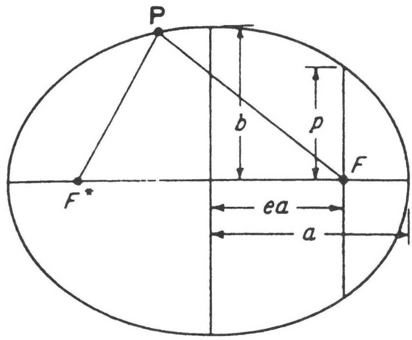

$PF + PF^{*} = 2a$

FIGURE 1. Ellipse

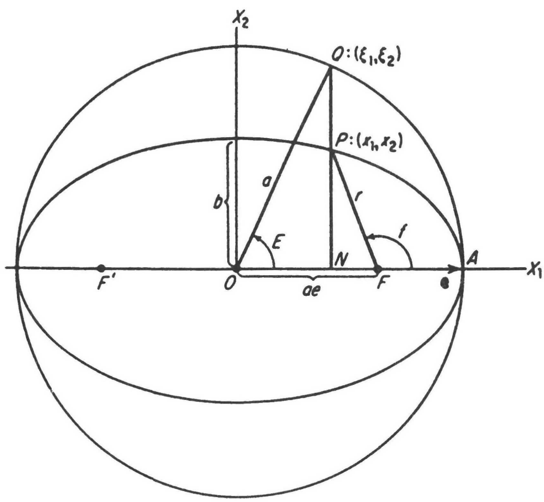

FIGURE 2. Elliptic Anomalies

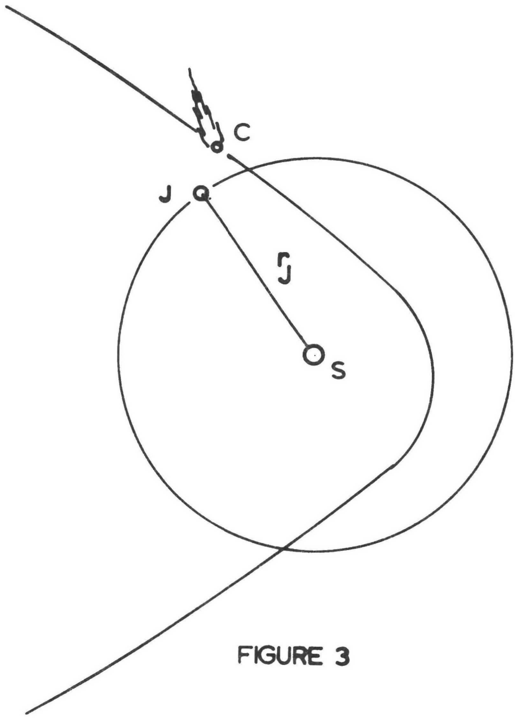

FIGURE 3

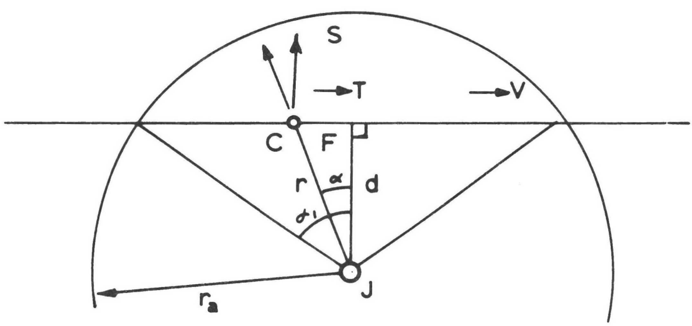

FIGURE 4

DISCUSSION

Other calculations for Comet Halley based on its dust trail give a history of 23,000 years (1) and a future of at least 100,000 years (2). Can you explain this discrepancy with your shorter time results?

The probability argument for the long period to short period transition leaves out the large number of comets assumed in the Oort cloud, perhaps trillions. Factored in, this gives dozens of possible new short period comets each year. Perhaps the main question is the existence of the Oort cloud, not probabilities.

The study depends heavily on the 1980s Halley mass loss, 20 tons/second. This loss is highly variable among comets, and also greatly decreases with each perihelion passage. The Halley figure is therefore of questionable significance.

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Don B. DeYoung, Ph.D.
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This is an excellent paper. I strongly recommend it.

Harold S. Slusher, Ph.D.
El Paso, Texas

Dr. Stillman re-investigates the implications of short cometary lifetimes for the age of the solar system. Unfortunately, his mass loss analysis is flawed by the assumption that it is inversely proportional to the square of the distance from the sun because it is dependent upon solar pressure. It appears rather that mass loss occurs in jets whose existence depends upon temperature, which may be stochastic in nature, and which do not occur beyond a certain distance from the sun. His estimates, however, are probably close enough to be illustrative. In his dynamic analysis, he neglects the theoretical "inner Oort cloud" which was postulated to overcome the difficulties of transition from long-period to short-period comets. The ultimate conclusions are not significantly changed by these oversights, but an analysis which included them would be more convincing.

Paul Steidl, M.S.
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CLOSURE

Dr. DeYoung refers to a short review by Maddux in Nature (vol.339,1989). This article discusses a recent Canadian study which relied on a chain of inferences and some radar-obtained dust measurements to deduce that Halley's comet has been present in the inner solar system for no more than 23,000 years. The number and magnitude of uncertainties present in that study hardly say anything about the interpretations one could draw from the computations presented in this paper which, after all, is primarily devoted to a forward look based on some scanty information now available. The inferred future of at least 100,000 years is at variance with much of the scientific literature, as well as the reported extrapolation here.

The Halley's comet mass loss figure is a weak reed on which to lean, but essentially it's all we've got at this time. Acknowledging that the constant mass loss rate (for a given radius) assumption is somewhat dubious I extended the scope of the numerical integration program to consider loss rate proportional to the surface area of the comet nucleus. With a variable loss rate it became necessary to integrate over the entire projected life of the comet, rather than the half-orbit examined in the body of the paper. After running the Apollo computer much longer we obtain the revised prediction of Halley's comet life of 25,169 years, compared to the more simplistic value of 16,273 years presented earlier. An increase in life of approximately 50% hardly affects the thrust of this paper.

The presence of ejecta in comet nuclei is well known. There are also observed variations in dynamic behavior (jumps) which can probably be attributed to unobserved jetting. As measurement capabilities are enhanced and more data are accumulated it is expected that much more rigorous (and satisfying) studies of comet durability can be conducted. Nevertheless it is safe to say that the basic conclusion of the study - namely the extinction of short-period comets in a very few thousand years - will in all likelihood remain little affected.

The Oort cloud is at present little more than a myth, although a very necessary myth, as indicated by the amount of space devoted to the concept by Sagan and Druyan. Even more dubious is a so-called inner Oort cloud mentioned by Mr. Steidl. The whole point of the exercise was to play some games with this idea, should it be granted any substance, which I doubt. Yes there are postulated to be perhaps trillions of comet nuclei in this entity, generally positioned at incredible distances from the sun. We did not examine the first half of the scenario, which is the generation of long period comets passing close enough to the sun to be affected by one of the larger planets. A subsequent study, it is hoped, will indicate that the number of potential long period comets is quite limited. Clearly the second portion, which I briefly addressed, is also worthy of much more attention.

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