

EIGENVALUE ANALYSIS OF THE MAGNETIC FIELD OF THE EARTH AND ITS IMPLICATIONS ON AGE AND FIELD REVERSALS

Kent R. Davey
School of Electrical Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0250

ABSTRACT

The temporal evolution of the geomagnetic field is analyzed using classical electromagnetics via an eigenvalue approach. The analysis points out the error of thinking of the Earth's field decay as a simple exponential decay, the decay constant being the equivalent inductance / resistance ratio of the Earth's core. In reality the field is characterized by a continuum of characteristic decay times (or eigenvalues). Each eigenvalue is associated with a unique shape. The total field is the sum of each eigenshape (or eigenvector) each decaying at a different rate. Since the weighting factors on each of these shapes is dictated by the initial field penetration of the core, it is likely that higher order eigenvectors will have larger weighting factors than the fundamental, more slowly decaying eigenshapes. Field reversals are an expected consequence of correctly analyzing the field transient.

The conclusions of this work are as follows. First, the age of the Earth's field, accounting for the whole continuum of characteristic eigenvalues, cannot be much more than 10,000 years old. Second, one or more field reversals can be expected in the first 1000 years of the field's existence. These reversals are caused by two sources. The first originates in the weighting of the individual eigenvectors as dictated by the initial core field. The second source is through eddy current induction in the ionosphere and/or vapor canopy.

The third conclusion is that motion effects in the core will not give added longevity to the field; the primary motion of the Earth is in the wrong direction to alter the eigenvalues. Secondary motion, i.e. precession, will only perturb the eigenvalues, and these perturbations have the wrong angular dependence around the globe to collectively couple to the main field.

INTRODUCTION

Geochronology is a difficult game to play since the experiment cannot be repeated. The magnetic field of the Earth is an enigma to both creationists and evolutionists. The evolutionist must appeal to dynamo theories [1] and motion effects [2] to support the long age he is committed to believe. The creationist must deal with apparent evidence for magnetic field reversals [3-5]; these explanations include turbulent movements in the core during the Noachic flood [6] and magnetostrictive effects. The intent of this article is to address from a theoretical perspective the following questions:

- (1) When and how does fluid core motion affect the characteristic decay of the geomagnetic field?
- (2) Are global field reversals predicted by classical field theory independent of fluid core motion?
- (3) How can an upper limit be placed on the age of the Earth if there are an infinite number of eigenvalues characterizing the decay of the field?

THEORY

This analysis springboards from the work of Thomas Barnes[7], which found its motivation from that of Horace Lamb[8]. The objective is to predict the temporal evolution of the magnetic field of a conducting sphere stressed by a magnetic field at time $t=0$. The geometry is shown in Fig. 1. .

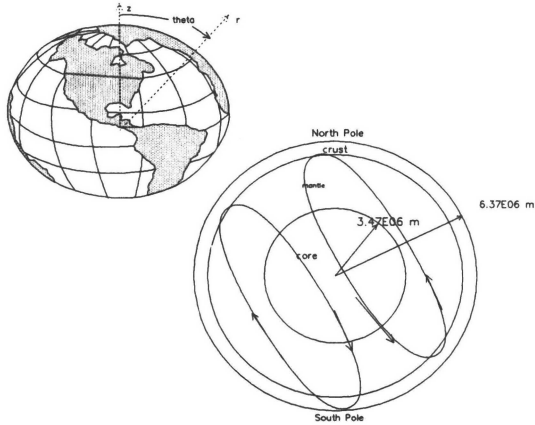


Fig. 1. Geometry of the problem to predict the field in and around the core of the Earth.

The core is thought to be primarily molten iron with a conductivity of 33,000 to 40,000 mho/m [9,10]. Because the temperature is well above the Curie point, we are quite safe in setting the permeability to $4\pi \cdot 10^{-7}$ Henry's/m. For this analysis we shall assume the currents in the core azimuthal or ϕ directed. As a first starting assumption we also assume the core is immersed in a uniform z directed field having a strength equal to that of a magnetic star, i.e., 100 Gauss. By uniformitarian geology, this would set an upper limit to the field, since the dust cloud (soon to be Earth) would have to be coalescing and cooling in close proximity to a large magnetic star to realize these conditions.

The analysis proceeds by solving Ampere and Faraday's equations for the magnetic (B) and electric fields (E),

$$\nabla \times \vec{H} = \sigma \vec{E} \quad (1)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (2)$$

subject to the requirement that the B field be divergence free, $\nabla \cdot \vec{B} = 0$. In this magneto-quasistatic regime, it is convenient to represent the B field as the curl of a vector potential A, i.e.,

$$\vec{B} = \nabla \times \vec{A}. \quad (3)$$

Combining equations (1-3) yields the result

$$\nabla^2 \vec{A} = \mu \sigma \frac{\partial \vec{A}}{\partial t} \quad (4)$$

Because the currents are assumed to be in the ϕ direction, the vector A has only a ϕ component. Following the lead of Barnes or Smythe [7,11], we can expand A as

$$A_{\phi} = A(\theta) \cdot A(r) \cdot \exp\left(-\frac{t}{\tau}\right) \quad (5)$$

In spherical coordinates, (4) becomes

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial A_\phi}{\partial r} \right) + \frac{1}{r^2 \sin(\theta)} \frac{\partial}{\partial \theta} \left(\sin(\theta) \frac{\partial A_\phi}{\partial \theta} \right) + \left(\frac{1}{L^2} - \frac{1}{r^2 \sin^2(\theta)} \right) A_\phi = 0. \quad (6)$$

where

$$L = \sqrt{\left(\frac{\tau}{\mu \sigma} \right)}.$$

The θ component of the field satisfies Legendre's equation and has the solution

$$A(\theta) = P_l^1(\cos(\theta)). \quad (7)$$

The only solution that is consistent with a dipole field such as we observe occurs when $l = 1$ yielding a θ field dependence of $\sin(\theta)$. With the substitution $r = zL$, the equation for the r component of A becomes

$$z^2 \frac{dA}{dz^2} + 2z \frac{dA}{dz} + (z^2 - 2)A = 0. \quad (8)$$

The solutions for this equation are spherical Bessel functions

$$j_1(z) = \sqrt{\left(\frac{\pi}{2z} \right)} J_{\frac{3}{2}}(z) \quad (9)$$

$$y_1(z) = \sqrt{\left(\frac{\pi}{2z} \right)} Y_{\frac{3}{2}}(z) \quad (10)$$

At this point, we make an important departure from previous developments in this area. The solution both inside and outside the core will be expressed as an infinite spectrum of eigenvalues L_s each with its own decay constant τ_s .

$$A_\phi(r < r_{core}) = \sum_s C_s j_1(z_s) \sin \theta e^{-\frac{t}{\tau_s}} \quad (11)$$

$$A_\phi(r > r_{core}) = \sum_s \frac{D_s}{r^2} \sin \theta e^{-\frac{t}{\tau_s}} \quad (12)$$

The boundary conditions that the normal and tangential components of the magnetic field be continuous pin down the allowable eigenvalues; these yield the two additional equations

$$\frac{D_s}{\alpha^2} = C_s j_1\left(\frac{\alpha}{L_s}\right) \quad (13)$$

$$C_s j_1\left(\frac{\alpha}{L_s}\right) + C_s r \frac{\partial j_1\left(\frac{\alpha}{L_s}\right)}{\partial r} = -\frac{D_s}{r^2}. \quad (14)$$

Note the variable α will be used henceforth for the core radius. These equations can be combined to give the requirement

$$j_0\left(\frac{\alpha}{L_s}\right) = 0 \quad (15)$$

or

$$\frac{\alpha}{L_s} = n\pi.$$

The solution for the total field follows by employing the orthogonality condition

$$\int_0^\alpha \left\{ \frac{r}{\alpha} j_1\left(\frac{r}{L_s}\right) \right\} dr = \frac{1}{2} \alpha \left\{ j_1\left(\frac{\alpha}{L_s}\right) \right\}^2 \quad (16)$$

If the initial core field is constant over the core having the value B_0 , then for any position r in the core at $t=0$ the following relationship must hold

$$\sum_s C_s j_1\left(\frac{r}{L_s}\right) = \frac{1}{2} \cdot B_0 \cdot r, \quad (17)$$

since

$$A_\phi = \frac{1}{2} \cdot r \sin \theta$$

corresponds to a uniform z directed B field. Carrying out the mathematics reveals the result

$$C_s = B_0 L_s \frac{j_2\left(\frac{\alpha}{L_s}\right)}{j_1^2\left(\frac{\alpha}{L_s}\right)} \quad (18)$$

$$D_s = \alpha^2 C_s j_1\left(\frac{\alpha}{L_s}\right) \quad (19)$$

where

$$\vec{B}_{out}(r > \alpha) = \frac{2}{r^3} \sum_s \cos \theta e^{-\frac{t}{\tau_s}} \hat{a}_r + \frac{1}{r^3} \sum_s D_s \sin \theta e^{-\frac{t}{\tau_s}} \hat{a}_\theta \quad (20)$$

RESULTS - UNIFORM INITIAL FIELD

Fig. 2. shows the θ directed surface field is predicted from (20) using 100 eigenvalues.

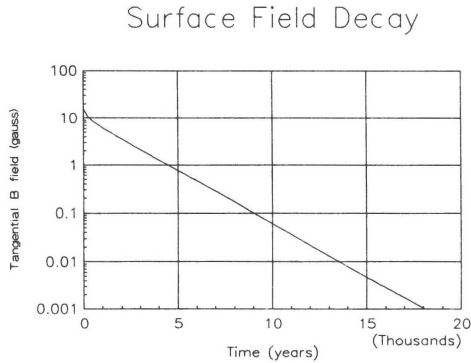


Fig. 2. Primary θ directed field on the surface of the Earth versus time in years.

These results are plotted keeping 100 eigenvalues; this number is adequate to represent the initial field to 4 decimal places. Upon close examination, the reader will note that the initial decay is not exponential (linear on a log scale). The initial decay is actually dictated by the higher order eigenvalues and decays more rapidly. An enlargement of the initial decay is shown in Fig. 3. .

The magnitude of the effect is determined by the weighting of the higher order eigenvalues, which follows from the initial conditions. Fig. 4. shows both the eigenvalue decay constants τ_s , and the weighting constants D_s .

Surface Field Decay

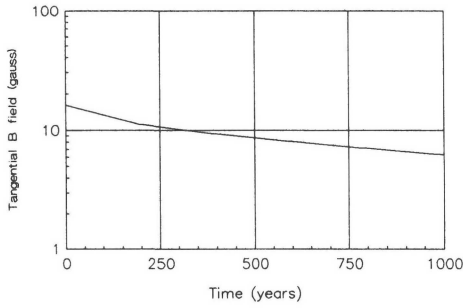


Fig. 3. Initial Field decay showing the more rapid decay influence of the higher order eigenvalues.

Eigenvalues and their Weights

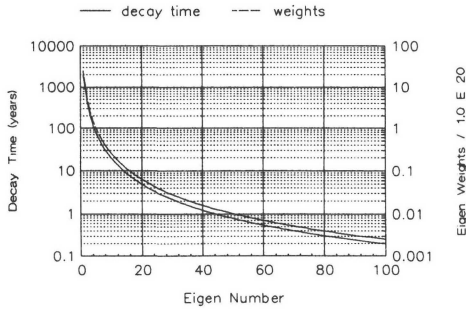


Fig. 4. Primary θ directed field on the surface of the Earth versus time in years.

With a core conductivity $\sigma = 4.04 \cdot 10^4$ mho/m the slowest (primary) time constant is $6.204 \cdot 10^{10}$ seconds or 1966.6 years; all other eigenvalues decay faster.

RESULTS FOR A VARIANT INITIAL FIELD

The fact that the weighting values monotonically decrease for the above case is happenstance. It is more likely that the initial field was not uniform. The original vector potential was specified in (17). Suppose this field is altered so the vector potential has a sinusoidal character in the core,

$$A_{\phi} = \frac{B_0}{2} r \cos\left(\frac{\pi r}{2a}\right) \sin\theta \tag{21}$$

Using the vector potential insures preservation of a divergence free B field. This vector potential and commensurate B field are shown in Fig. 5. .

To allow for any initial condition the weighting constants were performed numerically using Gauss Quadrature integration routines. The formula for finding the weighting constants of any vector potential having radial field dependence is

Variant Initial Field

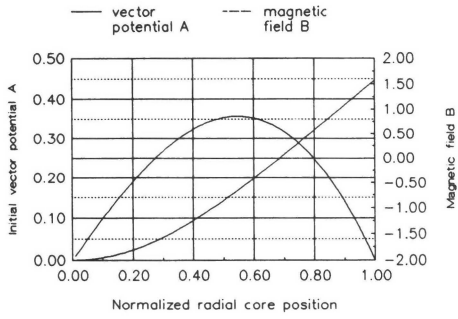


Fig. 5. Vector potential and commensurate tangential B field consistent with equation (21).

$$C_s \frac{a}{2} j_1^2\left(\frac{\alpha}{L_s}\right) = \int_0^a g(r) \left(\frac{r}{a}\right)^2 j_1\left(\frac{r}{L_s}\right) dr \quad (22)$$

The new decay constants and their weighting values are shown in Fig. 6. .

Eigenvalues and their Weights

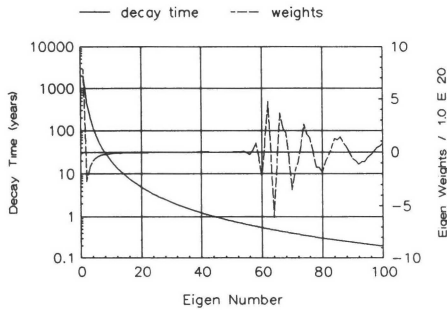


Fig. 6. Decay constants and eigenvector weightings for the variant field shape.

The resulting field decay is quite distinct from the constant field shape (Fig. 7.). As shown more clearly in the blowup shown in Fig. 8. , the initial field actually reverses in the first 200 years.

Surface Field Decay

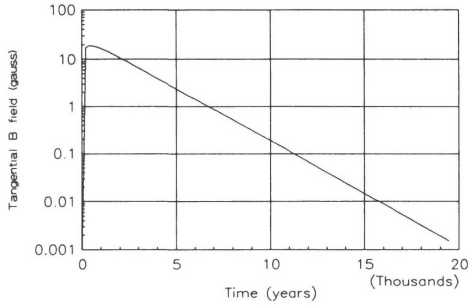


Fig. 7. Field decay with time for the variant field starting condition.

Surface Field Decay

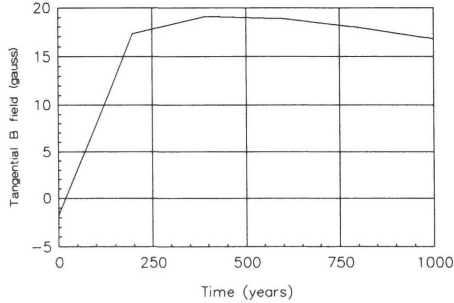


Fig. 8. Initial field decay shows a reversal of the field in the first 200 years.

RELEVANCE TO FIELD REVERSALS

It is impossible to know exactly what shape the initial field had. The important issue is there are numerous possibilities which produce higher order eigenvector weightings with magnitudes larger, and signs opposite to the more dominant earlier eigenvalues. When this happens, field reversals will occur. These reversals are in no way tied to core fluid motion.

A second mechanism by which the reversals can take place is through induction. About 10% of the Earth's field is external in origin [1,2], mostly through ionospheric currents. When the internal field decays it induces current in the atmosphere which attempts to oppose the decay. The amount of current is proportional to the rate of decay of the primary field. It is clear from Fig. 5 that some eigenvectors of the field are decaying hundreds of times faster than the fundamental present day decay during the initial period, but their weight factors C_s, D_s are not proportionally smaller. They will induce very large currents both in the ionosphere and in the pre-flood canopy which will yield large perturbation fields in multiple directions.

MOTION EFFECTS

A recent article in Earth Science [12] suggests that we may be due for another reversal of the Earth's field. The article revolves around the popular notion that fluid motion in the core has a profound affect both on the field decay and its orientation at any time in geological history. The mechanism of possible interaction as revealed in an eigenvalue analysis discloses serious flaws with these notions. In a moving frame the electric field E' is related to that when no motion is present as

$$\vec{E}' = \vec{E} + \vec{V} \times \vec{B} \quad (23)$$

where

\vec{v} = the velocity describing movement of the medium in the presence of an external field \vec{B}

Incorporating this into the equation describing A gives

$$\nabla^2 \vec{A} = \mu\sigma \frac{\partial \vec{A}}{\partial t} - \vec{v} \times \nabla \times \vec{A} \quad (24)$$

The last term on the right of (24) must be ϕ directed to have any influence at all. This immediately rules out any effect the predominant rotation (also ϕ directed) of the Earth might have. The fact that the Earth's axis is offset from the magnetic axis is no help; since the internal field is essentially homogeneous in the core, the $\vec{V} \times \vec{B}$ field will yield radially directed currents which will cancel side to side.

The precession of the Earth offers some hope at first. The precession is a θ directed movement; the last term of (24) would be

$$\omega_\theta r \frac{1}{r \sin(\theta)} \frac{\partial}{\partial \theta} (\sin \theta A) \quad (25)$$

where

$$\vec{v} = \omega_\theta r$$

It would be tempting to write (24) as

$$\nabla^2 A + \left(\frac{\mu\sigma}{\tau} + 2\omega_\theta \right) A = 0 \quad (26)$$

and then to assess the effects of on the rate of decay by noting that the quantity in parentheses would be the same (i.e., same eigenvalues). If the precession frequency were 200,000 years, the affect would be sizable were it not for one mistake. All but the last term in (24) have a $\sin(\theta)$ dependence; the $\cos(\theta)$ of the last term balances out over the globe. The only type of motion that will have a lasting affect on the dipole field of the Earth, yielding both a ϕ component for the last term in (24) and the appropriate θ dependence in space, is a radial velocity motion. It is hardly necessary to point out that Bernard induced thermal motion will be quite small indeed.

This analysis does not rule out transitory fluid effects as suggested by Humphreys [6], which have a $\cos(m\phi)$ dependence with m not equal to 0. These affects will definitely decay more quickly than those eigenvalues shown in Figs. 4 and 6. A localized turbulent eddy will surely give rise to a current eddy (the two eddy loops will interlink in fact), but the field will decay very rapidly because of the dissipatory nature of each. Unless the turbulence is very large these effects could be safely ignored.

CONCLUSIONS

The complete eigenvalue analysis offers new insight into how the Earth's field decays with time. Field reversals are expected not from complex internal core fluid motion, but initial field conditions. Regardless of these initial conditions, the eigenvalue decay times are known and set; the longest of these is only 1966 years. Because the decay constants τ_s are identical regardless of the initial field, it is difficult to even imagine how the Earth's field can last more than 20,000 years. Complex core turbulence may be responsible for perturbations in the field, but no long term field - fluid coupling appears plausible.

REFERENCES

1. H. Jeffreys, The Earth, 4th Ed., Cambridge Univ. Press, Cambridge, 1959.
2. J. Jacobs, R.D. Russel and W.J. Tuzo, Physics and Geology, McGraw Hill, New York, 1959.
3. J.G. Neigi and R.K. Tiwari, "Magnetic Field of the Earth involving its Dipole Field Aspect", Sci. AM, vol 250, p.70, March 1984.
4. D.R. Humphreys, "Has the Earth's Field ever flipped?", CRSQ, 25 (Dec. 1988).
5. D. Strangway, History of the Earth's Magnetic Field, New York, McGraw Hill, 168 pp, 1970.
6. D.R. Humphreys, "Reversals of the Earth's Magnetic Field During the Genesis Flood",

- Proceedings of the First Int'l Conference on Creationism, vol II, Pittsburgh, pp 113-126, 1986.
7. T.G. Barnes, Origin and Destiny of the Earth's Magnetic Field, ICR, El Cahan, CA 1973.
 8. H. Lamb, Phil. Trans, London, 174:519, 1813.
 9. T.G. Barnes, "Decay of the Earth's Magnetic Moment and the geochronological implications", CRSQ, 9, pp 222-230, Mar 1973.
 10. F.D. Stacey, "Electrical Resistivity of the Earth's Core", Earth and Planetary Science Letters, 3, pp 204-206, 1967.
 11. Smythe, Static and Dynamic Electricity, McGraw Hill, pp 378-380, 1967.
 12. S. Bannergy, "Is Reversal of Earth's Magnetic Poles Due?", Earth Science, vol 37, p 10, spring 1984.

DISCUSSION

This is an excellent theoretical paper on the decay of the earth's magnetic field. Of particular importance is Davey's solution in which he deduces, from an initial variant "magnetization" in the core of the earth, reversals of the earth's magnetic field in the early stage of the decay. It is the first rigorous theoretical solution of reversals which this reviewer has seen.

He accomplished his stated mission. It led to the following conclusions: "Field reversals are expected not from complex internal core fluid motion, but from initial conditions" (at creation). "Regardless of these initial conditions, the eigenvalue decay times are known and set: the longest is only 1966 years." That is strong support for a very young age. "Complex core turbulence may be responsible for perturbations in the field, but no long term field-coupling appears plausible."

Thomas G. Barnes, D.Sc.
El Paso, Texas

The eigenvalue approach Dr. Davey presents is a good start toward clarifying creationist analyses of the earth's magnetic field. There must be a factor missing from the first term of his final result in eq. 20—the first term has no dependence on the initial field B_0 . I think the missing factor is D_s . If so, then eq. 20 is simply a superposition of dipole fields of various decay times. A more general solution of the Maxwell equations for this situation, including higher-order spherical harmonic terms such as the quadrupole and octopole moments, would have been useful, because 10% of the present field is non-dipolar, and the past field even more so.

One good contribution of this paper is its clear demonstration that classical field theory can account for several global reversals. That was implicit in eq. 9 of my previous ICC paper, but I never spelled it out. Another important contribution is that the shape of the initial field at creation could excite enough of the higher decay modes (or in his terms, "eigenvector weightings") to produce some global reversals shortly after creation. I had not realized this was a possibility. It could explain some evidence for reversals in Precambrian strata. (Data in those strata are difficult to interpret because even though most Precambrian rocks were probably formed before the Flood, they may have been globally re-heated during the flood, thus acquiring their magnetizations during the reversals of the flood period.) However, Dr. Davey apparently does not realize that the disturbances I hypothesize in the core during the flood would strongly excite *all* the decay modes, not just the modes with non-zero values of m [last paragraph of second-to-last section]. This sets up a new set of initial conditions for the period after the flood, making his analysis even more relevant to the post-flood period than it is to the post-creation period.

It is also important to note that this paper's classical mechanism cannot account for the fifty or more rapid reversals which the evidence clearly indicates occurred during the Flood [Davey's ref. 4]. By the way, I think many (and perhaps most) of those reversals were *global*, a point I have not emphasized before this.

The second-to-last section of the paper attempts to analyze the effect of fluid motions. Unfortunately, the analysis falls far short of Dr. Davey's goals, for two reasons:

- 1) It ignores magnetohydrodynamic effects, in particular, that magnetic flux lines move with the fluid. This means that the vector potential A in eq. 24 interacts with the velocity v , with the result that eventually the magnetic flux looks like spaghetti instead of the simple poloidal (azimuthal A) field considered by the author.
- 2) While acknowledging that radial (up and down) fluid motions can induce the necessary azimuthal electric currents, the author dismisses the possibility of such motions with one assertion: "... Bernard [Benard?] induced thermal motion will be quite small indeed." I think he means by this that convection flows of the fluid are negligible, but he produces no justification for this opinion. One of my papers at this conference (1) shows that convection flows, far from being negligible, could have easily been the major cause of the rapid reversals during the flood. However, my paper does support the main point Dr. Davey intended to make in this section: "... motion effects in the core will not give added longevity to the field."

REFERENCES:

1) Humphreys, D.R., "Physical Mechanism for Reversals of the Earth's Magnetic Field During the Flood", SECOND INTERNATIONAL CONFERENCE ON CREATIONISM, Creation Science Fellowship, Pittsburgh, PA, 1990, in press.

D. Russell Humphreys, Ph.D.
Albuquerque, New Mexico

The work presented by Dr. Davey is a straightforward and insightful analysis of the behavior of the earth's geomagnetic field in the absence of source terms. A plausible argument based on the presence of higher-order, rapidly decaying eigenmodes, is presented to explain early-time reversal of this field, a phenomenon that is often attributed to fluctuations in the earth's dynamo with a characteristic timescale of millions of years. I can find no fault with Dr. Davey's analysis nor with the implication that field-reversal does not demand an old-earth hypothesis. Nevertheless, Dr. Davey alleges to have proven what he has merely (though, perhaps correctly) assumed, namely the absence of source term. In his section entitled "Motion Effects," he dismisses the coupling of rotational energy to magnetic energy based on the assumption that the currents are always predominately θ -directed. This assumption, though intuitive, hardly leads to the proof implied in the conclusions. I would urge the author to restate his conclusion to more accurately reflect the assumptions as well as the insights of the paper.

Thomas W. Hussey, Ph.D.
Albuquerque, New Mexico

CLOSURE

The purpose of this paper was to establish an upper limit age index on the primary portion of the Earth's field which is dipolar in shape and show that classical field theory addresses multiple field reversals. This was the reason for not analyzing the higher order field components.

The statement about the analysis not being able to handle multiple reversals is quite erroneous. Depending on the initial shape of the field, the weighting constants for each and every multipole field is established at time $t=0$.

The reader may argue that if you carry out the calculations, you'll not be able to see any more reversals. It must be stressed that it is not a matter of carrying out the calculations further in time. The key issue is the shape of the field at the start of the transient. The reader is referred to the classical work of Smythe [10] if further clarification is needed. To amplify the point however, consider the initial field shape shown in Fig. 9:

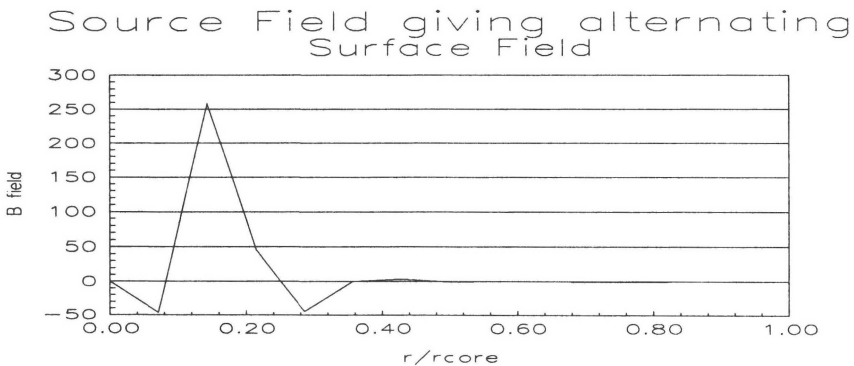


Fig. 9. Variant Initial core field shape emphasizing multiple reversals.

The amplitude is not as important as the decomposition of the shape into each of the eigenvector components. Because of the assumed azimuthal dependence ($m=1$) all components must be dipolar in nature, but all components decay at a different rate. Fig. 9 happens to be a shape that weights

the higher order components more heavily than the dominant decay components. The commensurate field decay is shown in Fig. 10. Little emphasis should be given to the magnitude of the initial field or the magnitude of the variations of the decay field; the initial field was set up arbitrarily to yield unity variation reversals in Fig. 10:

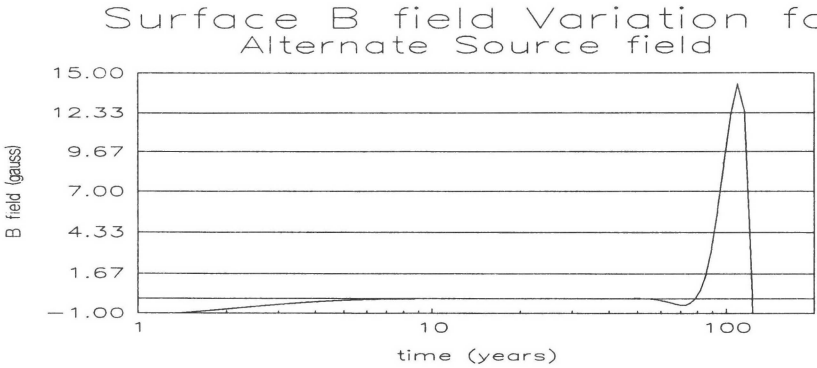


Fig. 10. Field decay commensurate with an initial shape such as Fig. 9.

This is not all that is impressive in terms of the number of reversals until one looks more closely at the "quiet zone" between 10 and 50 years. Fig. 11 shows a blowup of this region with the multiple reversals clearly displayed.

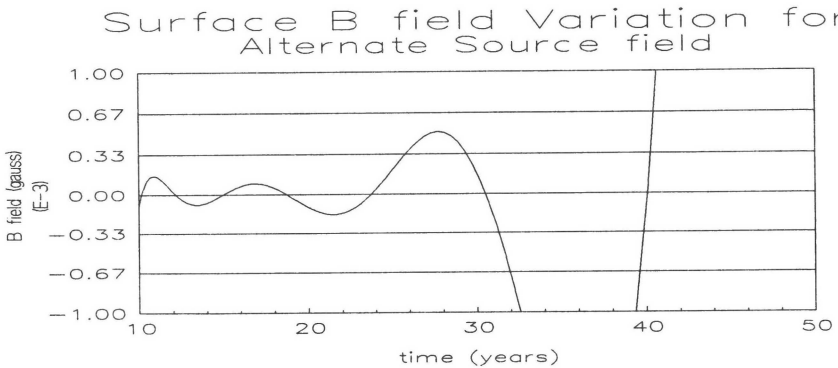


Fig. 11. Field decay commensurate with an initial shape such as Fig. 9 showing a blow up of the faster field reversal region.

By including shapes that encourage higher order eigenvalues, the reversal period can be made very small. In fact there is no theoretical limit to how close these reversals can be in time or to how many there can be. The longest time period is of course fixed by geometry and composition and cannot be altered; it is the long time constant that sets an upper limit to the age of the Earth.

Time $t=0$ is not necessarily the origin of the Earth as Dr. Humphreys points out. In this conference there was a paper discussing the breakup and sinking of the Earth's lithosphere near the event of the flood. Suggest an event, were it to occur, would constitute a sizable perturbation of the core conductivity. This is one mechanism for exciting all eigenmodes, and setting up a global reversal field such as Fig. 11.

Comments on Motion Effects of Drs. Humphreys and Hussey

I've added additional comments to the section on fluid motion in the paper itself which clear up some of the confusion. The real issue is whether there are $(v \times B)$ terms having a ϕ

component and a $\sin(\theta)$. Granted there is radial fluid motion of the core, but $\text{del}(v)$ must be essentially zero (to the extent of density variations). For radial motion to couple long term to the dipole field of the Earth, the radial velocity must have no θ dependence. The radial motion one expects to have will have a $\sin(m\theta)$ or $\cos(m\theta)$ dependence, which will not couple long term to the dipole field of the Earth.

Kent R. Davey, Ph.D.

